Modeling spot, forward and option prices of several commodities: a regime switching approach

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   - Proposition

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   - Introduction of the stochastic discount factor
   - Historical and Risk Neutral dynamics
   - Derivatives pricing

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Introduction: commodity modeling in EDF

- Short term (1 day → 2 weeks) prediction
- Market comprehension
- Mid term risk management
  - Gross energy margin prediction
  - Risk measurement
  - Hedging
- Pricing
  - Valuation of Production assets
  - Valuation of flexibilities in supply contracts
- Investment decision
What is needed

- **Portfolio exposition**
  - Power and fuels spot prices
  - Power and fuels forward products

- **What is needed**
  - forward products modeling
  - spot prices modeling
  - calibration procedures
  - for several commodities

- **Proposition: an econometric approach**
  - as an alternative to the classic continuous-time approach
  - capturing the main properties of spot prices (spikes, link between commodities)
  - defining the needed concepts to obtain a risk-neutral probability (the concept of the stochastic discount factor)
Econometric approach
The Stochastic Discount Factor and the Risk-Neutral probability

Fundamental assumptions:
1. Existence of prices
2. Absence of arbitrage opportunity (static)
3. Absence of arbitrage opportunity (dynamic)

Consequence [Bertholon et al. 2008, Monfort & Féron, 2012]:
- Existence of a stochastic discount factor (SDF) $M_{t,T}$

\[ M_{t,T} > 0 \]
\[ M_{t,T} = M_{t,t+1}M_{t+1,t+2} \ldots M_{T-1,T} \]

- Existence of a risk-neutral probability

\[ \mathbb{E}_Q^t [X_T | w_t] = \mathbb{E}_Q^t [X_T] = \mathbb{E}_P^t \left[ \frac{M_{t,T}}{\mathbb{E}_t [M_{t,T}]} X_T \right] \]

- Pricing of a derivative whose payoff is $g(S_T)$:

\[ p_t [g(S_T)] = \mathbb{E}_Q^t \left[ g(S_T) e^{-r(T-t)} \right] \]
Modeling point of view

- **3 elements**
  - Historical dynamics \((\mathbb{P})\)
  - Stochastic Discount Factor \((M_{t,t+1})\)
  - Risk-Neutral dynamics \((\mathbb{Q})\)

Two elements have to be defined, the third is deduced.

- **Incomplete market** : There is an infinity of SDF consistent with observed prices.

- **Restriction** (*a priori*) of the acceptable set of SDF with an affine exponential form to obtain closed-form formulas of pricing:

\[
M_{t,t+1} = \exp \left\{ \alpha_t(w_t) + \beta_t(w_t)w_{t+1} \right\}
\]

with \(w_t\) the all set of random variables at date \(t\).

- **This restriction must, at least, respect AOA properties** (i.e. some internal consistency conditions).
Our modeling choices

- Regime switching VAR(m) for the historical dynamics
  - with eventually non-homogeneous Markov Chain
  - Capturing stylized facts of the spot prices (spikes, changes of volatility...)
  - Using classic estimation procedures (Hamilton filter)

- Regime switching VAR(m) for the risk-Neutral dynamics
  - with homogeneous Markov Chain
  - Allowing closed-form (or quasi-closed form) formulas for forward prices and European options
  - Using classic estimation procedures (Kalman, extended-Kalman filters)
Modeling choice
Discrete time of the economy \((t = 1, 2, \ldots, T)\)

Information at date \(t\)
- Spot prices of \(N\) commodities, denoted by \((S_{1,t}, \ldots, S_{N,t})'\)
- Convenience yields \((\delta_{1,t}, \ldots, \delta_{N,t})'\) linking the forward price \(F_n(t, t + 1)\) to the spot price \(S_{n,t}\) [Schwartz, 1997]

\[
F_n(t, t + 1) = S_{n,t} e^{r_t - \delta_{n,t}}
\]

Remark: for Power spot prices, there will be no associated convenience yield

- The risk free rate \(r_t\)
- Endogenous qualitative variable \(z_t\), valued in \((e_1, \ldots, e_K)\)

Whole set of information at date \(t\)
\[
\mathbf{w}_t = (S_{1,t}, \ldots, S_{N,t}, \delta_{1,t}, \ldots, \delta_{N,t}, r_t, z_t)'\]
Historical dynamics

Notations

- Residual spot price logarithm \( \ln(S_{n,t}) = \nu_{n,t} + s_{n,t}, \ n = 1, \ldots, N. \)
- \( x_t = (s_{1,t}, \ldots, s_{N,t}, \delta_{1,t}, \ldots, \delta_{N,t}, r_t)' \)

Switching regime \textit{VAR(1)} on the residual noises:

\[
x_{t+1} = M_1 z_{t+1} + M_0 z_t + \Phi x_t + \Sigma_{1/2} (z_{t+1}, z_t) \varepsilon_{t+1}
\]

\( \varepsilon_{t+1} \) being a standard Gaussian white noise process of size \( 2N + 1 \), \( \Sigma_{1/2} (z_{t+1}, z_t) \) a symmetric positive definite matrix function

Markov chain on \( z_t \)

\[
P(z_{t+1} = e_j | z_t = e_i, s_t) = \pi_{ijt}
\]

\( z_t \) a state variable which takes \( K \) values corresponding to \( K \) regimes characterized by mean \( \mu_i \) variance \( \sigma_i^2 \) (ex: Nominal regime, high volatility regimes, spikes regime)
Risk-neutral dynamics

Proposition 1

Assuming the historical dynamics described by (1) and (2), if the stochastic discount factor has the following form:

\[ M_{t,t+1} = \exp \left\{ -r_t - \frac{1}{2} \alpha'(Z_{t+1}, Z_t, x_t)\alpha(Z_{t+1}, Z_t, x_t) + \alpha'(Z_{t+1}, Z_t, x_t)\varepsilon_{t+1} + \beta'(Z_t, x_t)Z_{t+1} \right\} \]

with:

\[ \beta(e_i, x_t) = \left( \log \frac{\tilde{\pi}_{i1}}{\pi_{i1}(e_1|e_i, x_t)}, \ldots, \log \frac{\tilde{\pi}_{iK}}{\pi_{iK}(e_K|e_i, x_t)} \right)' \]

\[ \alpha(Z_{t+1}, Z_t, x_t) = \Sigma^{-1/2}(Z_{t+1}, Z_t) \left[ (\Phi - \Phi)x_t + (\tilde{M}_1 - M_1)Z_{t+1} + (\tilde{M}_0 - M_0)Z_t \right] \]

then the Risk-Neutral dynamics is a regime-switching VAR(1) process:

\[ \tilde{x}_{t+1} = \tilde{M}_1'Z_{t+1} + \tilde{M}_0Z_t + \Phi x_t + \Sigma^{1/2}(Z_{t+1}, Z_t)\varepsilon_{t+1} \]  (3)

with a homogeneous Markov chain on \( Z_t \)

\[ \mathbb{P}(Z_{t+1} = e_j|x_t, Z_t = e_i) = \tilde{\pi}_{ij} \]  (4)
Pricing forwards

- **Pricing of a unitary forward price of the \( n \)th commodity** (the simple case with no seasonality part \( s_{n,t} = \ln S_{n,t} \) and \( \tilde{x}_t = x_t \))

\[
 F_n(t, T) = \mathbb{E}^Q_t [S_{n,T}] = \mathbb{E}^Q_t [\exp \{ \tilde{s}_{n,T} \}] = \mathbb{E}^Q_t [\exp \{ e'_n \tilde{w}_T \}]
\]

with \( \tilde{w}_t = (\tilde{s}_{1,t}, \ldots, \tilde{s}_{N,t}, \tilde{\delta}_{1,t}, \ldots, \tilde{\delta}_{N,t}, r_t, z_t)' = (\tilde{x}_t, z_t)' \)

- **Recurrence formula on Laplace transforms**

**Proposition 2** [Monfort & Féron, 2012]

If \( \tilde{w}_t \) follows the Risk-neutral dynamics defined by (3) and (4), we have:

\[
 \mathbb{E}^Q_t [\exp \{ e'_n \tilde{w}_T \}] = \exp \{ c'_{n,h} \tilde{w}_t \}
\]

where \( h = T - t \) and \( c_h \) are defined by recurrence:

\[
 c_{n,1} = a(e_n) \\
 c_{n,h} = a(c_{n,h-1})
\]
Internal consistency conditions

For any storable commodity, the unitary forward price $F(t, t + 1)$ can be obtained by two different ways:

- from the Risk-neutral dynamics (Proposition 2)
  \[
  F_n(t, t + 1) = \exp\{a(e_n)'\tilde{w}_t\}
  \]

- from the historical dynamics
  \[
  F_n(t, t + 1) = S_{n,t} \exp\{r_t - \delta_t\}
  \]

This leads to constraints in the recurrence function $a$ and then in the Risk-Neutral parameters (i.e. **internal consistency conditions**).
Possible extensions

- Seasonality parts (may be different in the historical and the risk-neutral probability)
- Increasing the auto-regression order to get $\text{VAR}(m)$ ($m$ can be different in the historical and risk-neutral dynamics)

**Forward product** price (in the case of non-storable commodity), with a delivery period $\theta$

$$F_{n}^\text{theor}(t, T, \theta, \tilde{p}) = \frac{1}{\theta} \sum_{T' \in [T; T+\theta]} F_{n}(t, T, \tilde{p})$$

the unitary forward price $F_{n}(t, T)$ being a function of the risk-neutral dynamics parameters $\tilde{p}$, the latent variables $z_t$ and $\delta_{n,t}$, and the observed $s_{n,t}$.

**Pricing European options** (Call on spot, Call on forward, Spread options) : using the Truncated Laplace transform [Monfort & Féron, 2012]
Historical dynamics: variable \((s_t, z_t)\)

\[
x_{t+1} = Mz_{t+1} + \Phi(x_t - Mz_t) + \Sigma^{1/2}(z_{t+1}, z_t)\varepsilon_{t+1}
\]

\[P(z_t = e_j | z_{t-1} = e_i) = \pi_{ij}\]

Risk-neutral dynamics:

\[
x_{t+1} = \tilde{M}_1'z_{t+1} + \tilde{M}_0z_t + \tilde{\Phi}x_t + \Sigma^{1/2}(z_{t+1}, z_t)\varepsilon_{t+1}
\]

\[P(z_{t+1} = e_j | x_t, z_t = e_i) = \tilde{\pi}_{ij}\]

Main issue for the estimation of observed spot prices and forward products

- Discrete latent variable \(z_t\)
- Continuous latent variable \(\delta_{n,t}\)
Estimation: proposed strategy

**Estimation procedure**

1. **Estimate the historical seasonalities** \( \nu_{n,t} \)

2. **Initialize the convenience yields** \( \delta_{n,t} \) by inverting the formula:

   \[
   F_n(t, T) = S_{n,t} \exp \{(r_t - \delta_{n,t})(T - t)\}
   \]

3. **Estimate the historical parameters on observed spot prices by Maximum Likelihood criterion using a Kitagawa-Hamilton filter (which also gives an estimation of \( z_t \))**

4. **Estimate the Risk-neutral dynamics on the \( N_f \) observed forward products by a non-linear least-square criterion**

   \[
   J(\tilde{p}) = \sum_{t=1}^{T_f} \sum_{i=1}^{N_f} \left[ \log F_{obs}(t, T_i, \theta_i) - \log F_{theor}(t, T_i, \theta_i, \tilde{p}) \right]^2
   \]

   using a (extended) Kalman filter (which also gives an estimation of \( \delta_{n,t} \))

5. **return to step 3 until a given stoping criterion**
The case with two commodities, $K=2$, $m=1$

- Historical data From 2010/01/01 to 2012/12/31
- UK Power (non-storable) spot prices, NBP Gas (storable) spot prices, EONIA (1day) interest rate

UK Power forwards: 1MAH $\rightarrow$ 3MAH, 1QAH $\rightarrow$ 4QAH, 1SAH $\rightarrow$ 6SAH

NBP Gas forwards: 1MAH $\rightarrow$ 3MAH, 1QAH $\rightarrow$ 4QAH, 1SAH $\rightarrow$ 6SAH
Results on forward reconstruction (not forecasting!)

(a) Power 1-Month-ahead
(b) Power 1-Quarter-ahead
(c) Power 1-Season-ahead
(d) Power 6-Season-ahead

Figure: Results of forward reconstruction: realized forward price (red line) versus reconstructed forward prices (blue line) for the 3 years of calibration.
Results on forward reconstruction (not forecasting!)

(a) Gas 1-Month-ahead

(b) Gas 1-Quarter-ahead

(c) Gas 1-Season-ahead

(d) Gas 6-Season-ahead

Figure: Results of forward reconstruction: realized forward price (red line) versus reconstructed forward prices (blue line) for the 3 years of calibration.
Conclusion and perspectives
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**Conclusion**
- Main contribution: propose an econometric approach for joint spot and forward prices, allowing quasi explicit formulas for European options
- Promising results in the Electricity spot prices (capturing spikes), forward and options [Monfort & Féron, 2012]
- Promising reconstruction results in the multivariate case

**Perspectives**
- The real case of multiple commodities (including Carbon for spark spread options)
- Improve the model and estimation procedure (to capture spikes)
  - for estimating a spike regime
  - for studying the estimation procedure (robustness...)

[12th Workshop on Stochastic Models, Statistics and Their Applications Finance Conference - Wroclaw - 2015]
References


