Modeling and forecasting electricity loads: A comparison

Adam Misiorek

Institute of Power Systems Automation

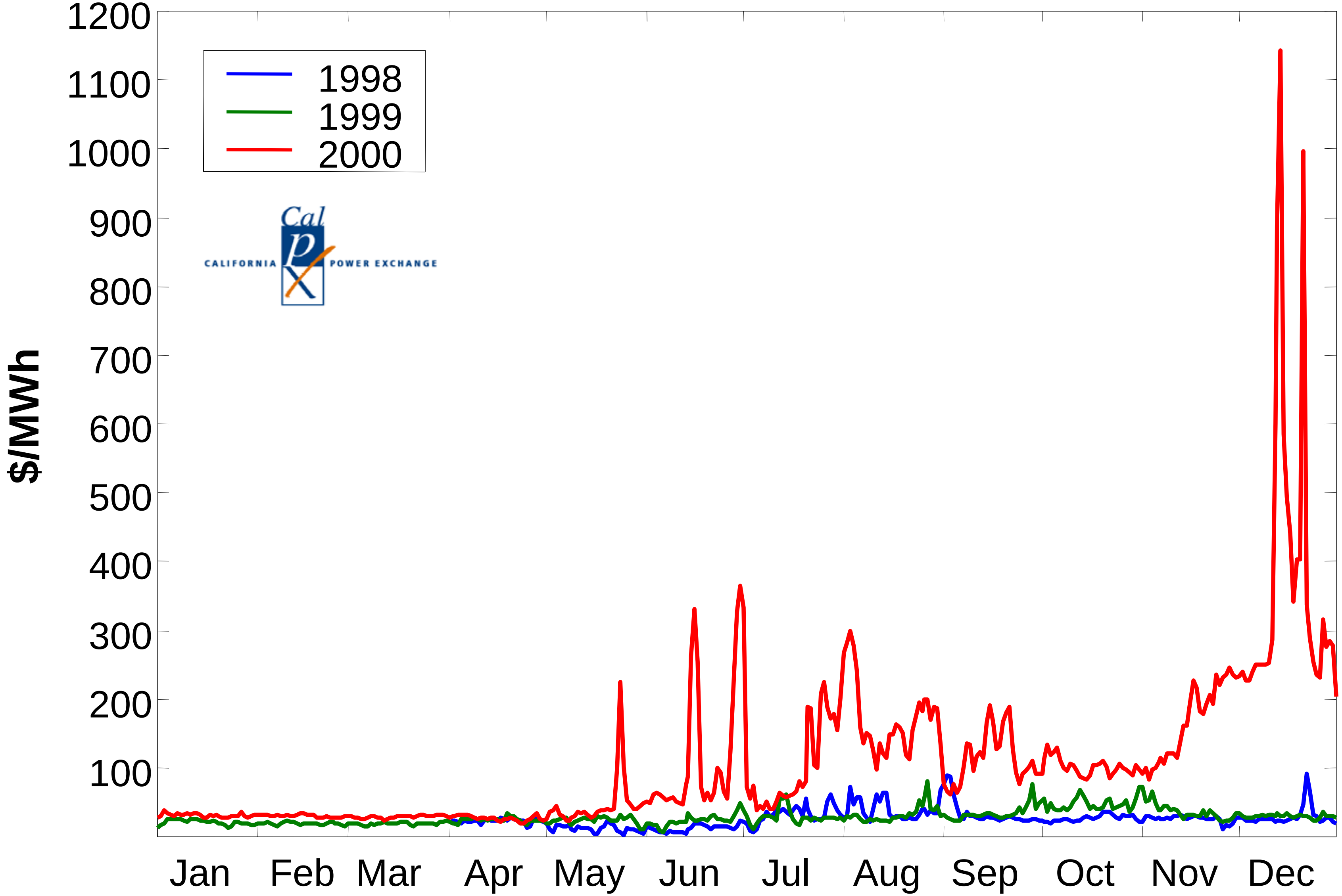
Rafał Weron*

Hugo Steinhaus Center

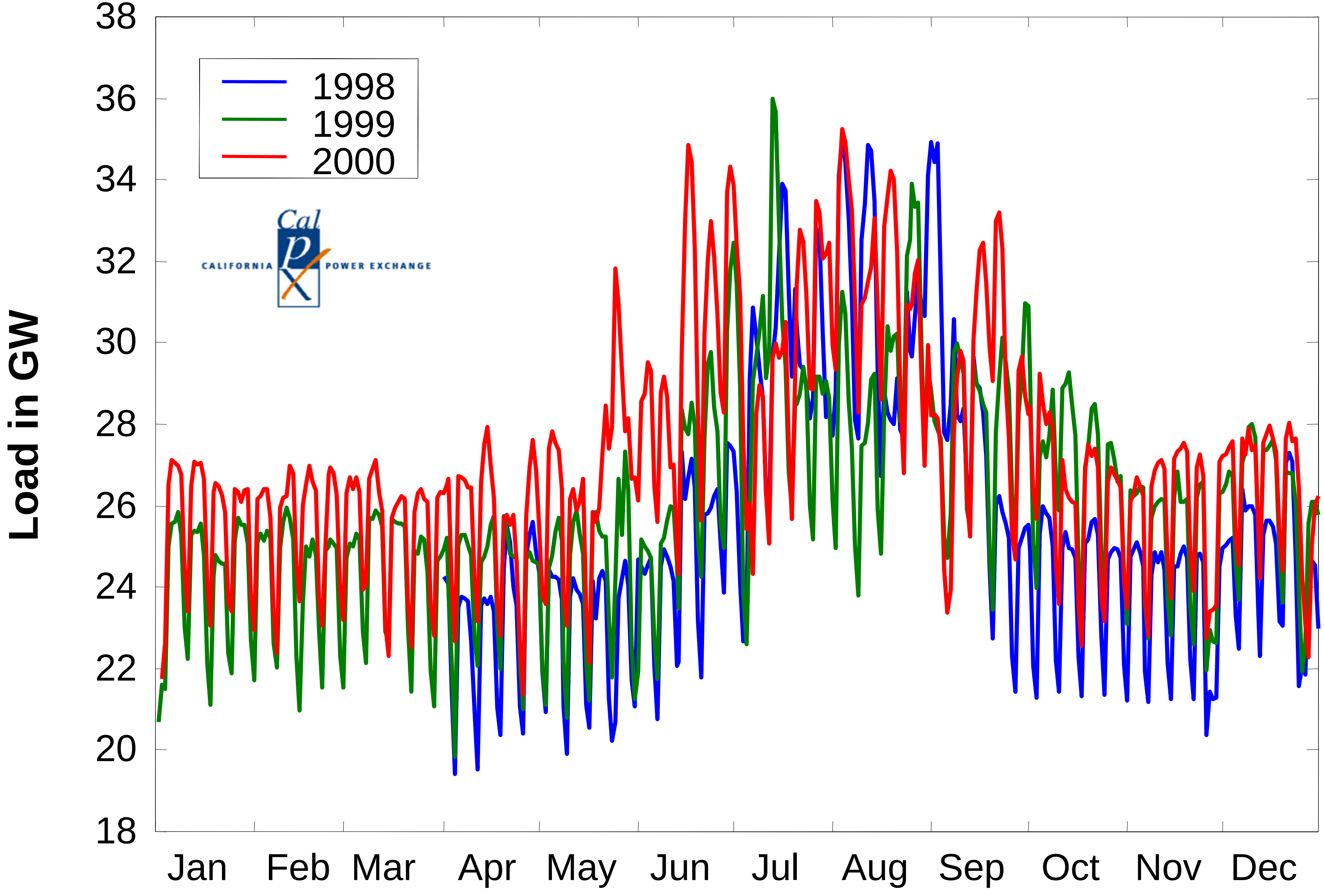
Analyzed data

- California power market system-wide load from the period 1.4.1998 – 31.12.2001
- Due to the daily cycle we analyze daily load
- 1999-2000 data used for calibration (2 full cycles), 2001 data used for out-of-sample testing

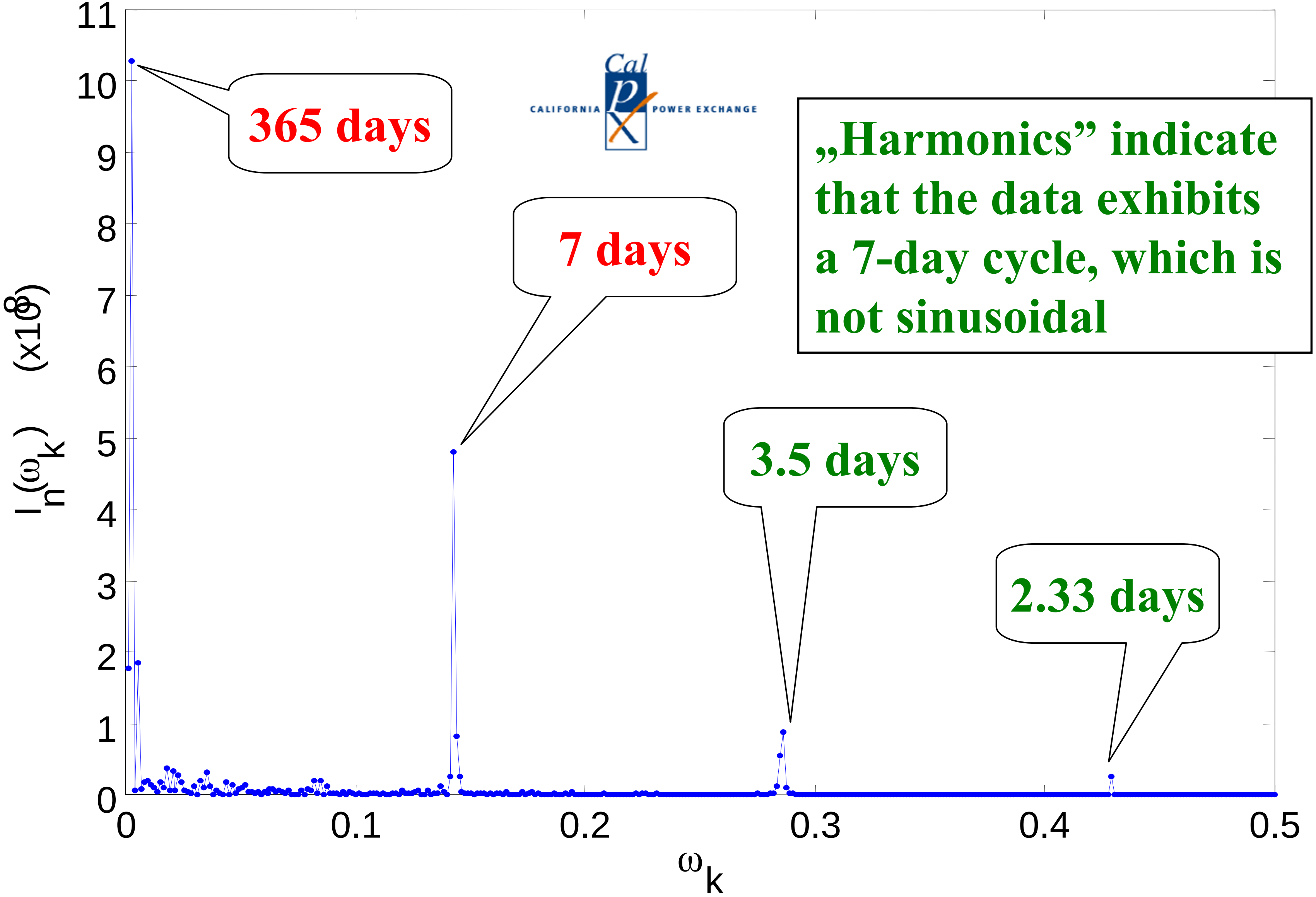
Average daily CalPX system electricity prices



Average daily system-wide load in California



Periodogram of the average daily load since 1.4.98 until 31.12.00



General statistical approach

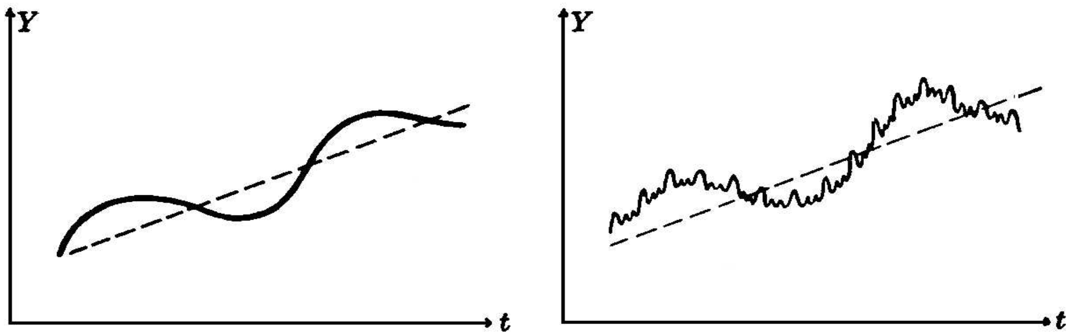
- Load Z_t is modeled as a sum of two components:
  - deterministic (or seasonal) X_t
  - stochastic Y_t
- The stochastic component $Y_t = Z_t - X_t$
  - AR, ARMAX, FARIMA, GARCH, FIGARCH, etc.
  - ARMA $\left(p, q, \phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q, \sigma^2\right)$

$$Y_t - \phi_1 Y_{t-1} - \ldots - \phi_p Y_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$$

$$\{\varepsilon_t\} \sim \mathrm{iid}(0; \sigma^2)$$

Seasonality reduction techniques

- Differencing
  - e.g. $X_t = Z_t - Z_{t\text{-}7}$
- Fitting a sinusoid or a sum of sinusoids



- Moving average method
- Seasonal volatility technique

Model A - Differencing

- Seasonal component

$$X_t = \frac{1}{N}\sum_{i=1}^{N} Z_{t-iT} + \frac{1}{M}\sum_{j=1}^{M} Z_{t-jD} - \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M} Z_{t-iT-jD}$$

where

- T = 168 hours (i.e. one week)
- D = 24 hour (i.e. one day)
- N = 4 or 5 is the number of weeks used for calibration
- M = 7 is the number of days in a week

Model B
- Moving average method

- For a vector $\{x_1, ..., x_{730}\}$ calculate a 7-day moving average $m_t = (x_{t-3} + ... + x_{t+3})/7$
- For each k = 1, ..., 7 calculate mean w_k of deviations $(x_{k+7j} - m_{k+7j})$ and rescale it so that w_k sum to zero
- The time series with eliminated weekly cycle is given by $\mathbf{x_t - w_t}$
- Unfortunately this method cannot be used to remove the annual cycle since data covers only two full cycles

Model B
- Seasonal volatility method

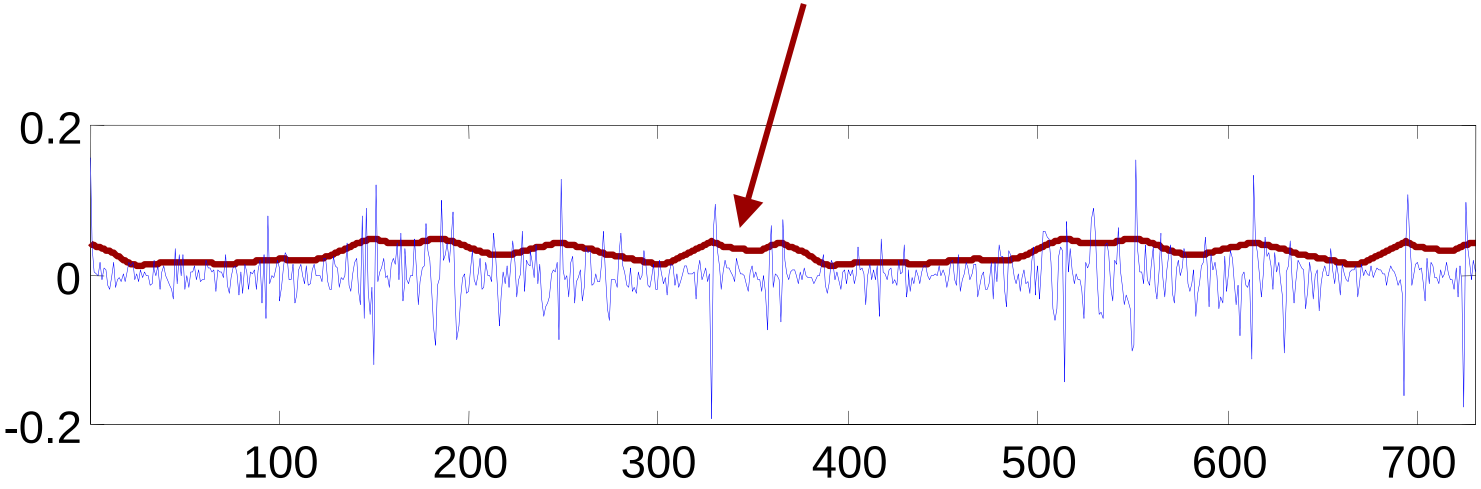
- Calculate a 25-day rolling volatility for the vector $\{x_1, \ldots, x_{730}\}$

$$v_t = \sqrt{\frac{1}{24}\sum_{i=0}^{24}\left(R_{t+i} - \overline{R}_t\right)^2}$$

- Calculate the average annual volatility $\sigma_t = (v_t^{1999} + v_t^{2000})/2$
- Smooth the volatility by taking a 25-day moving average of σ_t

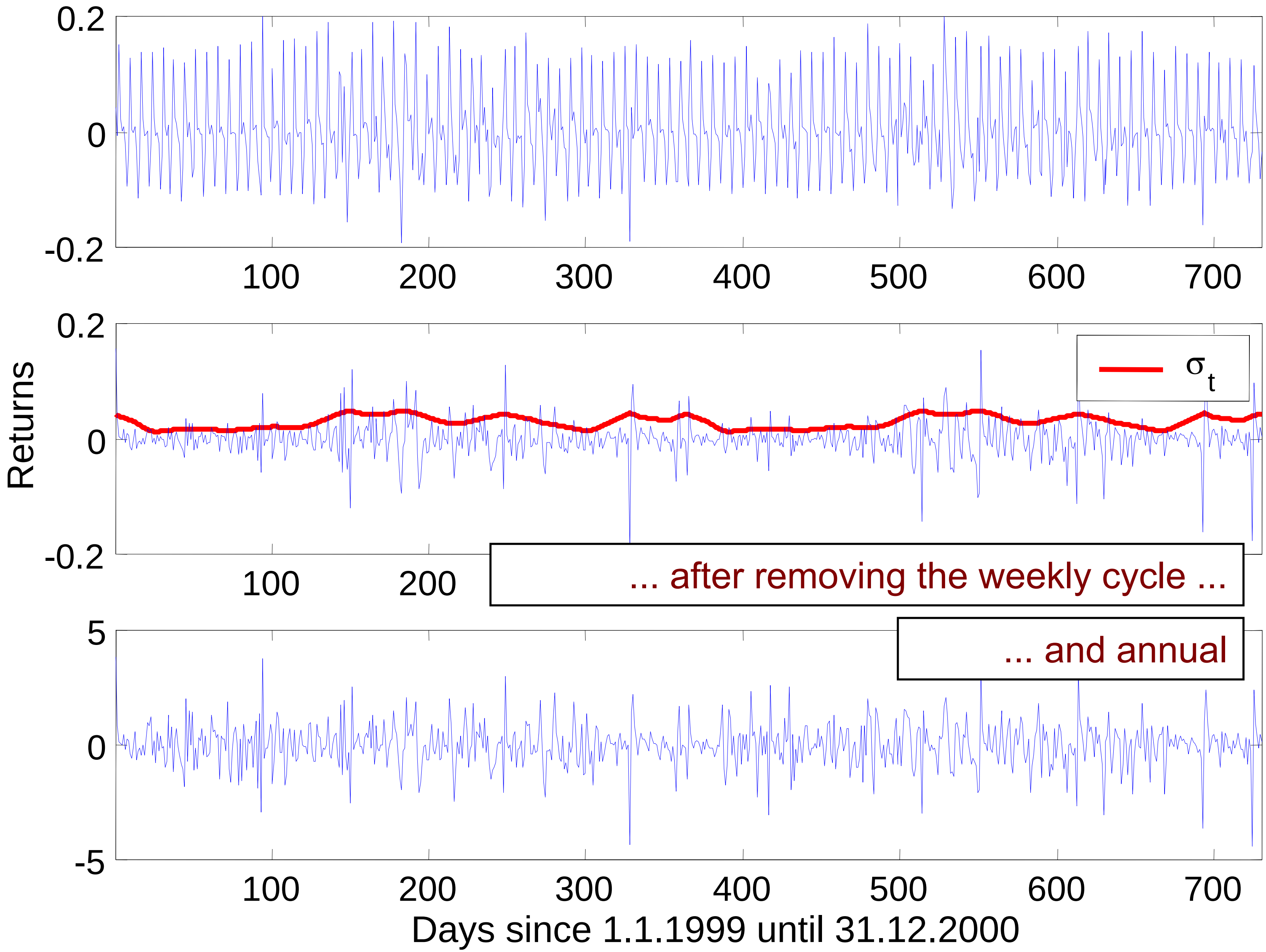
Seasonal volatility method cont.

- Finally rescale the returns by dividing them by the smoothed annual volatility $\overline{\sigma}_t$

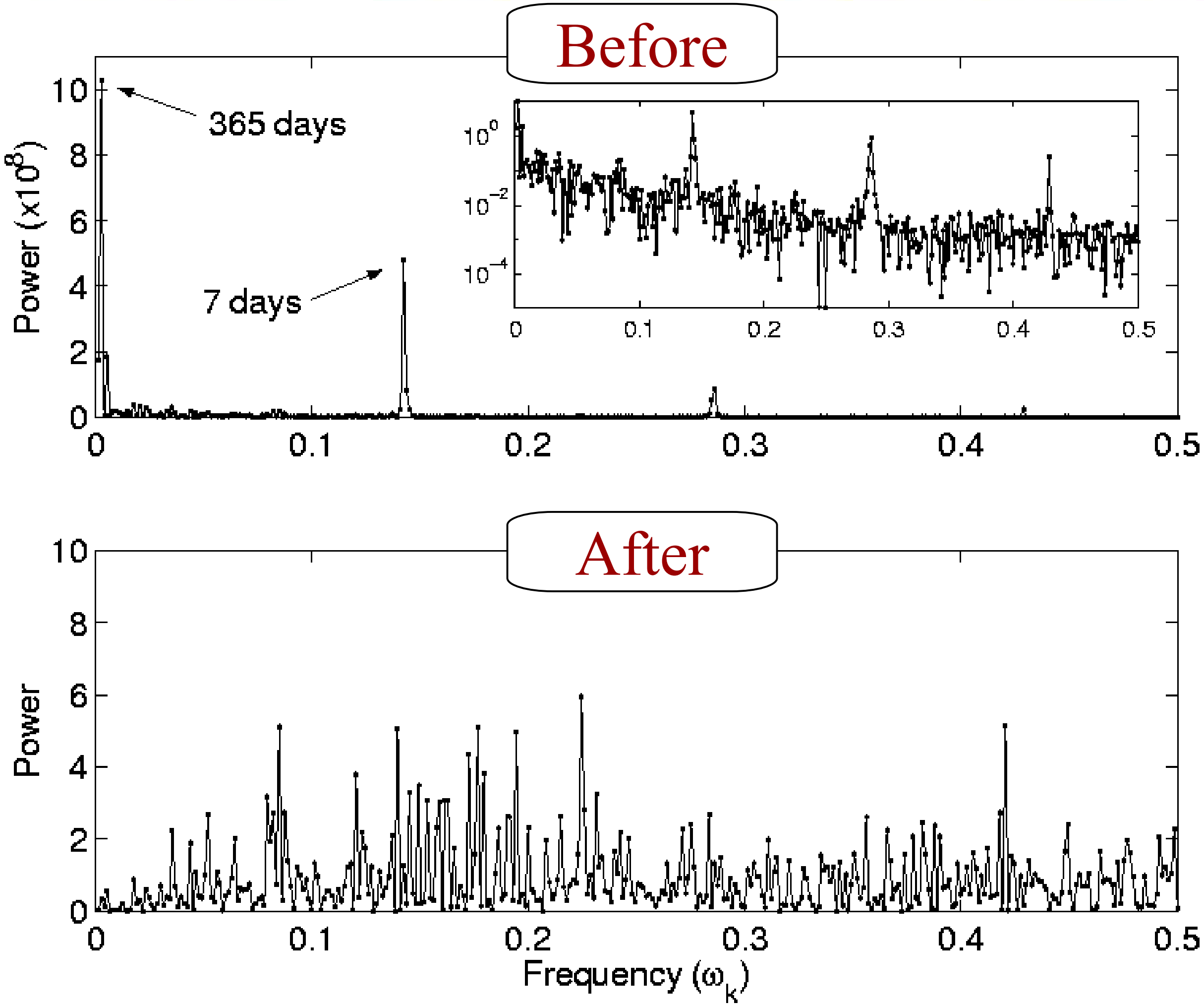


- The obtained time series does not exhibit the annual cycle

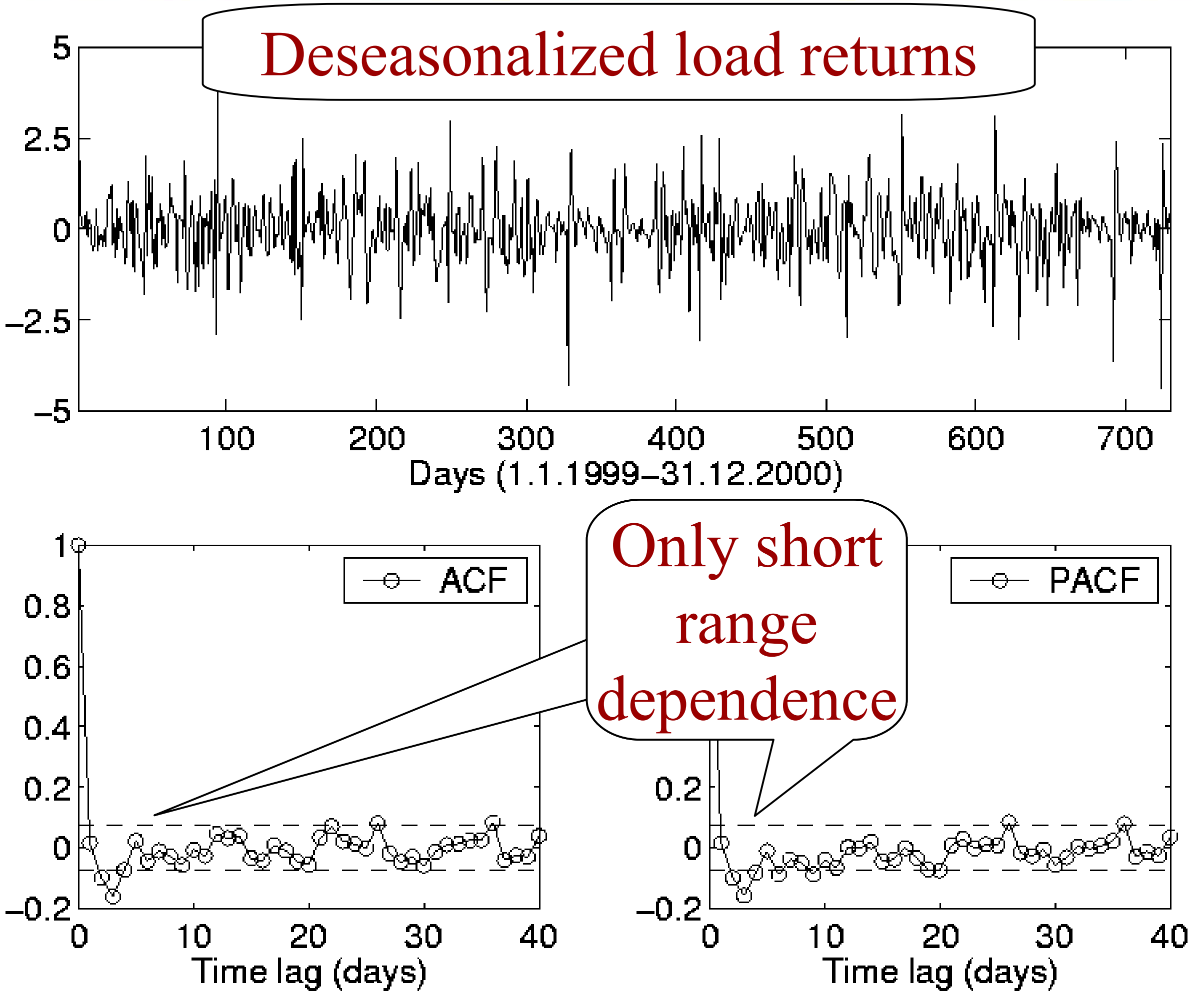
Returns of daily load ...



Periodogram before and after seasonality reduction



Deseasonalized load returns can be modeled by ARMA time series



Stochastic components

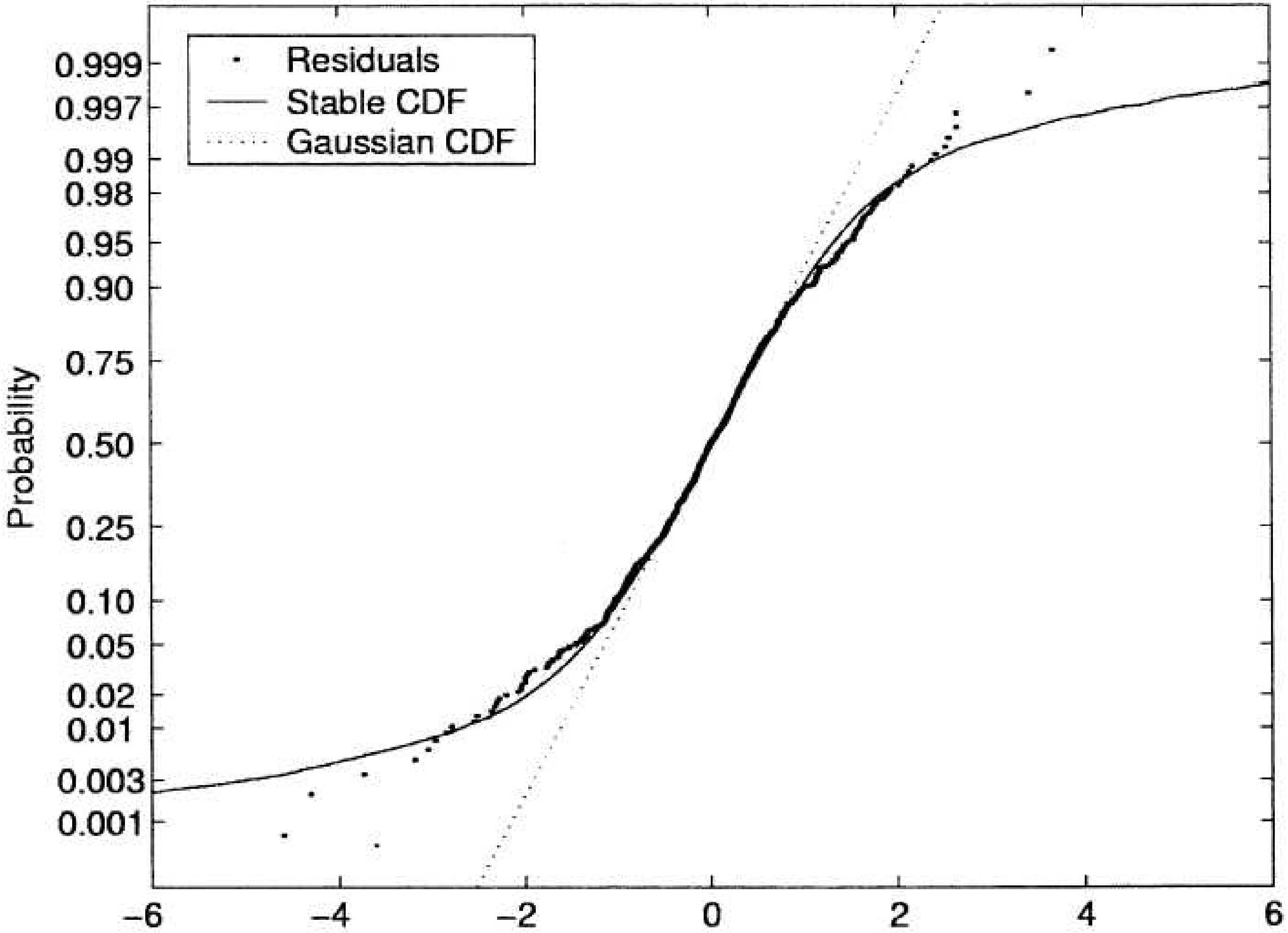
- Model A
  - ARMA(3,2)
- Model B

Table 1
AICC values for the best AR, MA and ARMA models

Model class	Best model	AICC value	Model class	Best model	AICC value
AR(·)	AR(11)	1965.226	MA(·)	MA(4)	1959.299
ARMA(1, ·)	**ARMA(1, 6)**	**1956.294**	ARMA(2, ·)	ARMA(2, 3)	1960.312
ARMA(3, ·)	ARMA(3, 3)	1962.037	ARMA(4, ·)	ARMA(4, 4)	1963.028
ARMA(5, ·)	ARMA(5, 3)	1962.045	ARMA(6, ·)	ARMA(6, 2)	1966.060
ARMA(7, ·)	ARMA(7, 3)	1967.511	ARMA(8, ·)	ARMA(8, 1)	1967.844
ARMA(9, ·)	ARMA(9, 1)	1965.556	ARMA(10, ·)	ARMA(10, 1)	1967.628
ARMA(11, ·)	ARMA(11, 1)	1968.005			

"·" denotes an integer from the interval [1,11].

ARMA models with hyperbolic noise



ARMA models with hyperbolic noise cont.

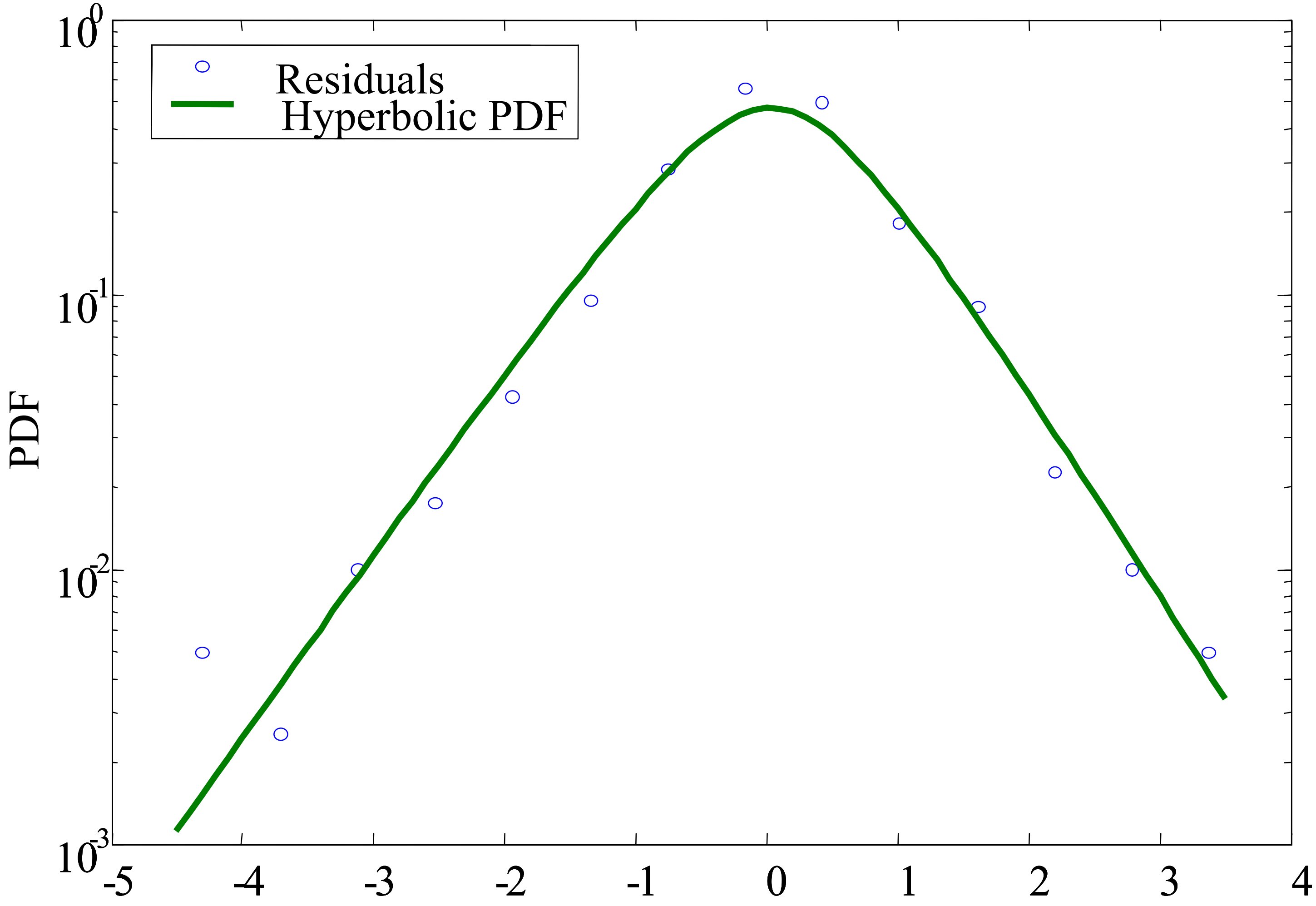
$$f(x;\alpha,\beta,\delta,\mu)$$

$$=\frac{\sqrt{\alpha^2-\beta^2}}{2\alpha\delta K_1(\delta\sqrt{\alpha^2-\beta^2})}\exp\{-\alpha\sqrt{\delta^2+(x-\mu)^2}$$

$$+\beta(x-\mu)\}, \tag{13}$$

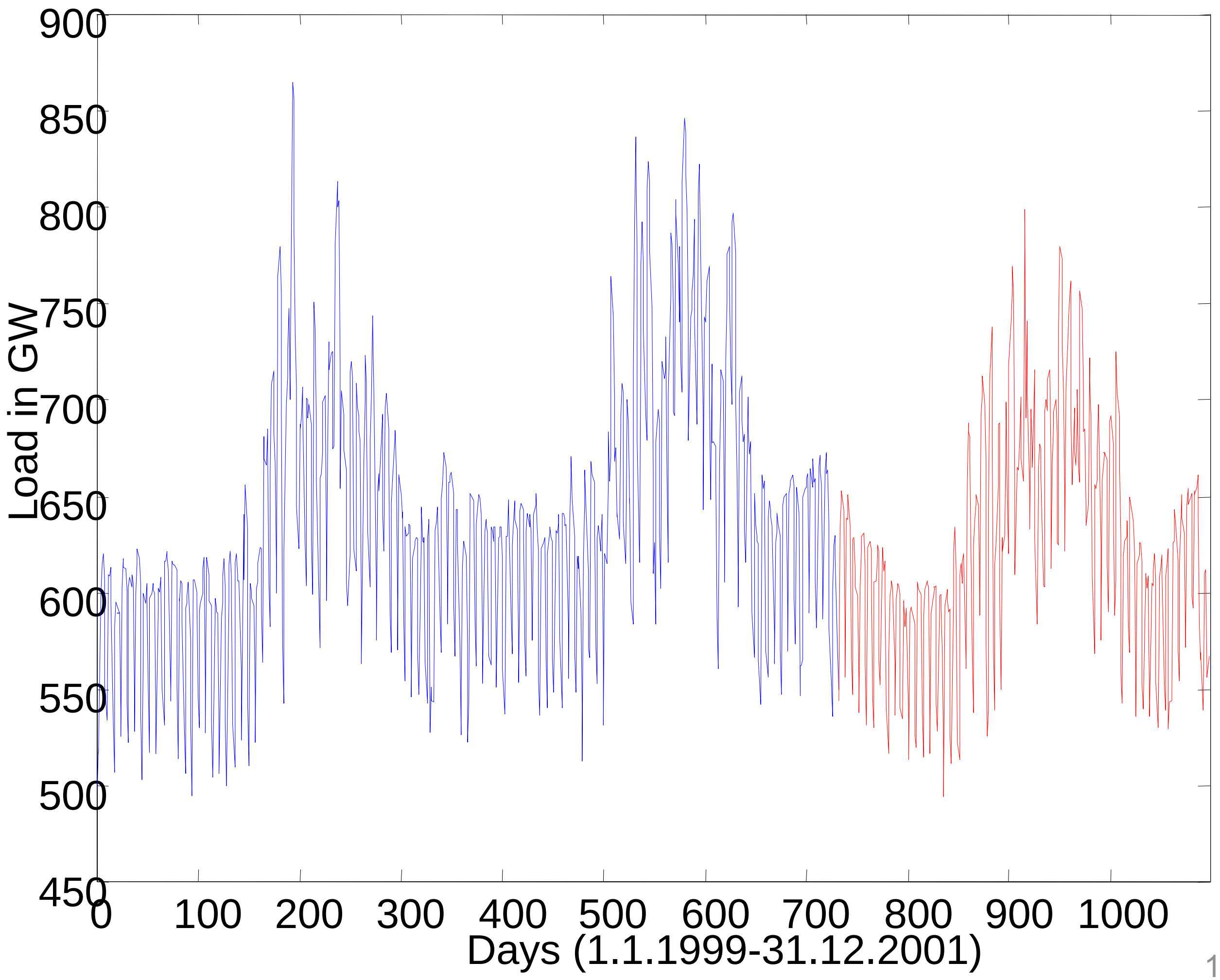
where $\delta > 0$ is the scale parameter, $\mu \in R$ is the location parameter and $0 \leqslant |\beta| < \alpha$. The latter two parameters—α and β—determine the shape, with α being responsible for the steepness and β for the skewness.

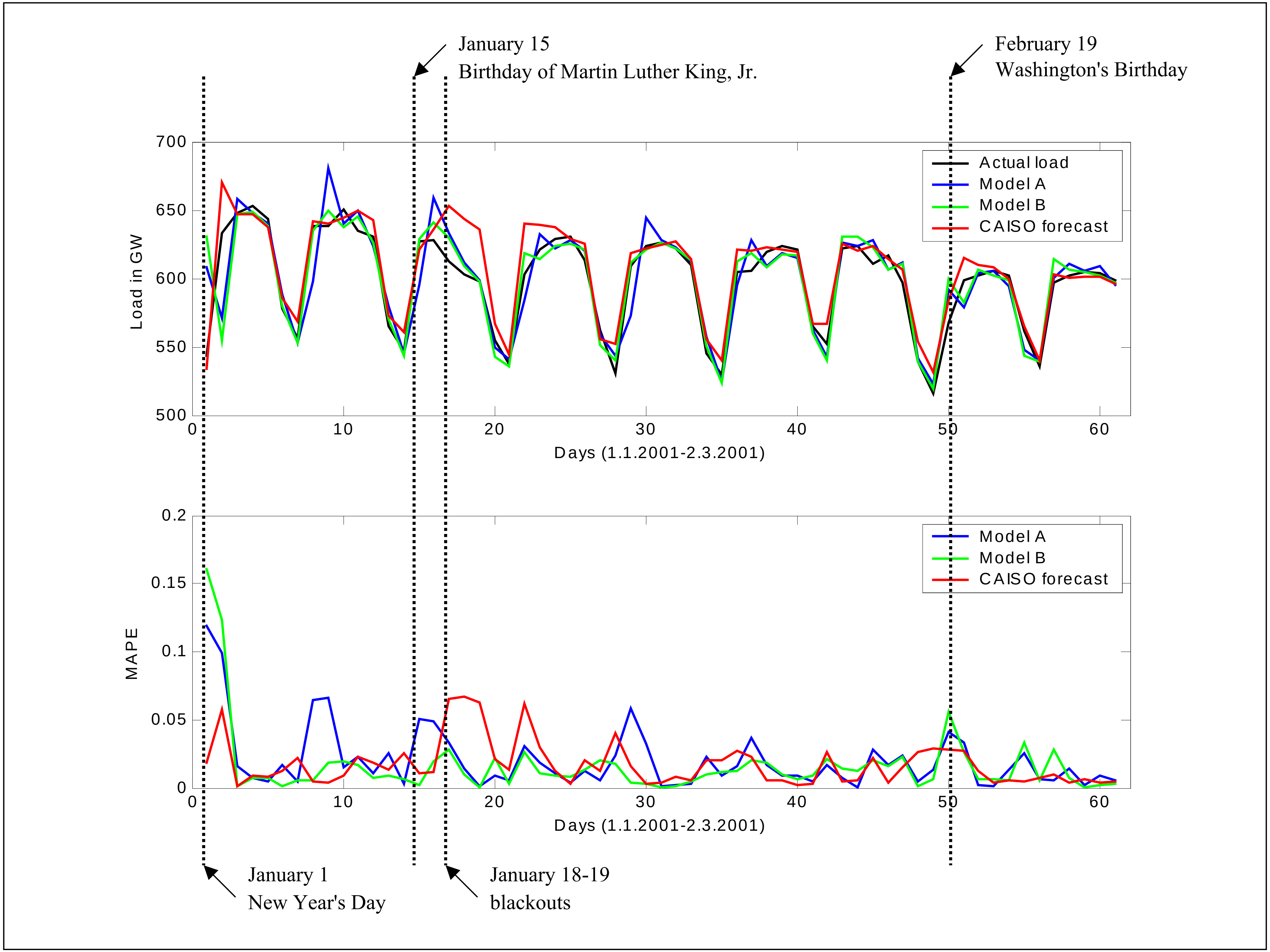
ARMA models
with hyperbolic noise cont.

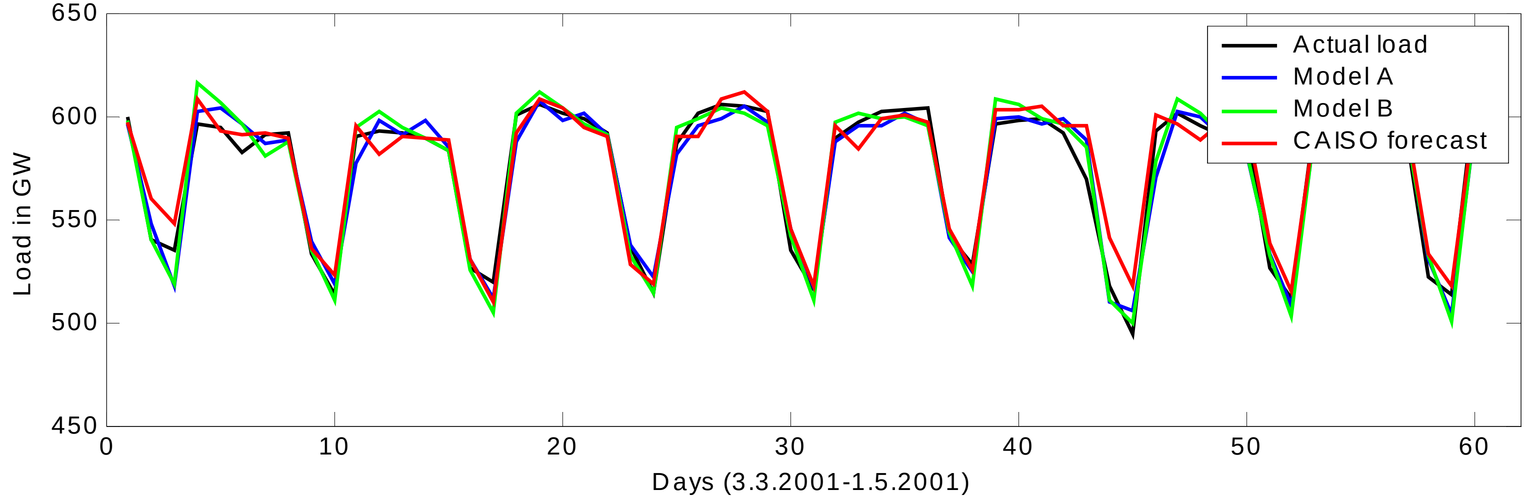


Out-of-sample tests

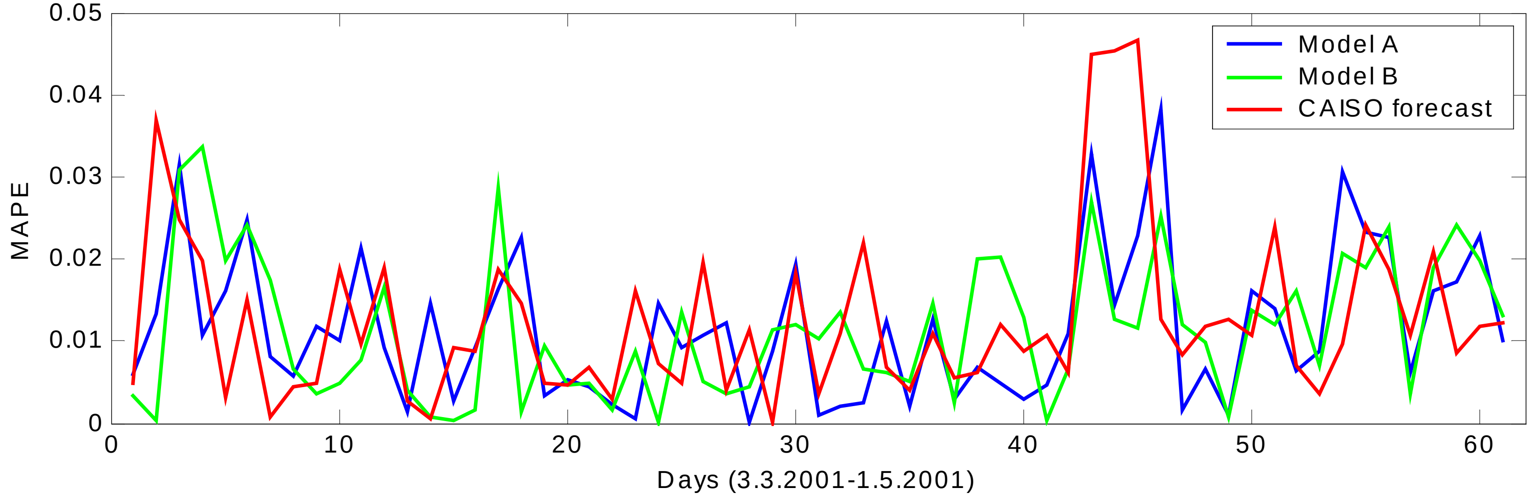
- Day-ahead predictions
- Adaptive scheme



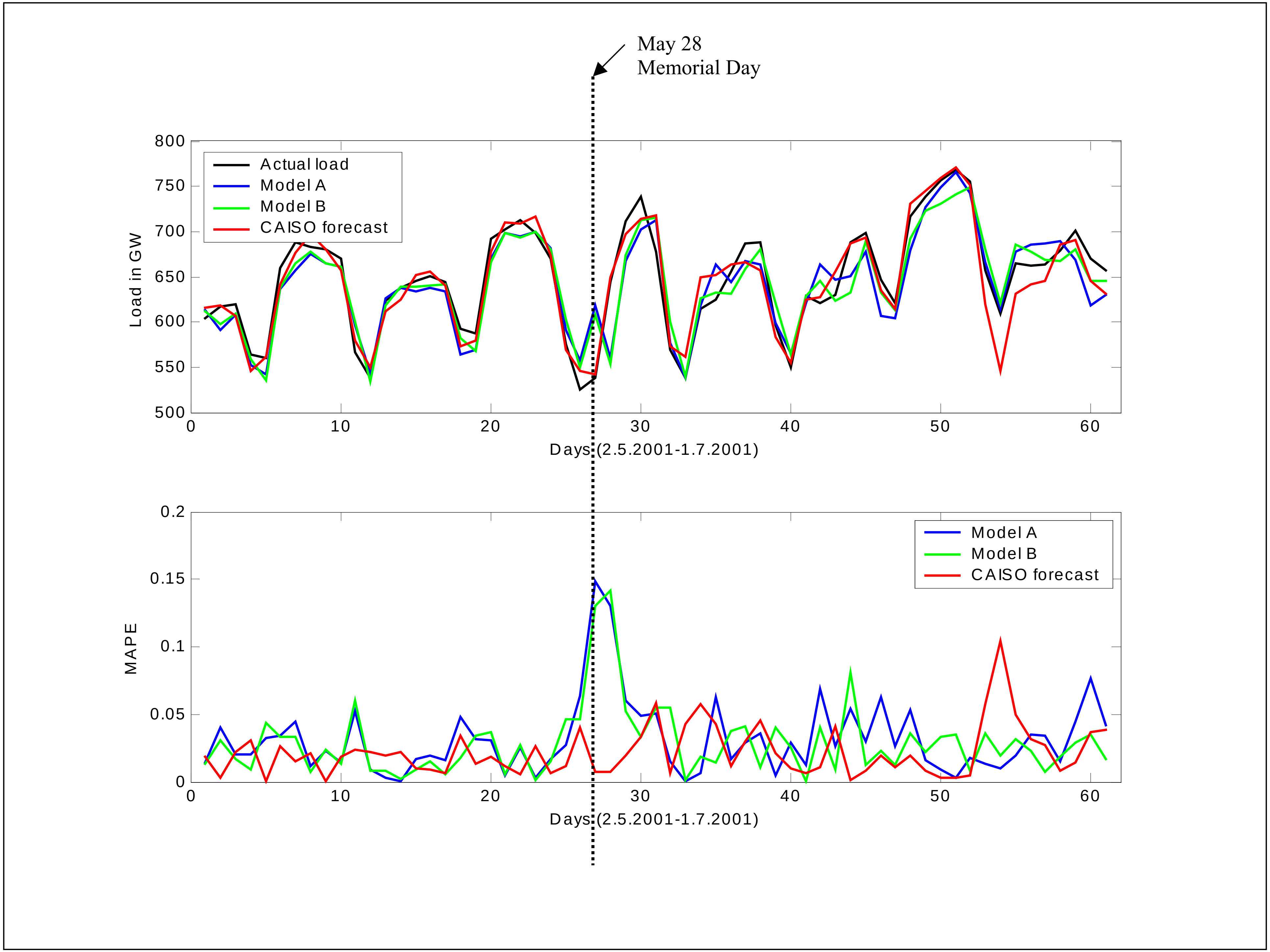

January 15
Birthday of Martin Luther King, Jr.
February 19
Washington's Birthday
Load in GW
700
650
600
550
500
Actual load
Model A
Model B
CAISO forecast
0
10
20
30
40
50
60
Days (1.1.2001-2.3.2001)
MAPE
0.2
0.15
0.1
0.05
0
Model A
Model B
CAISO forecast
Days (1.1.2001-2.3.2001)
January 1
New Year's Day
January 18-19
blackouts



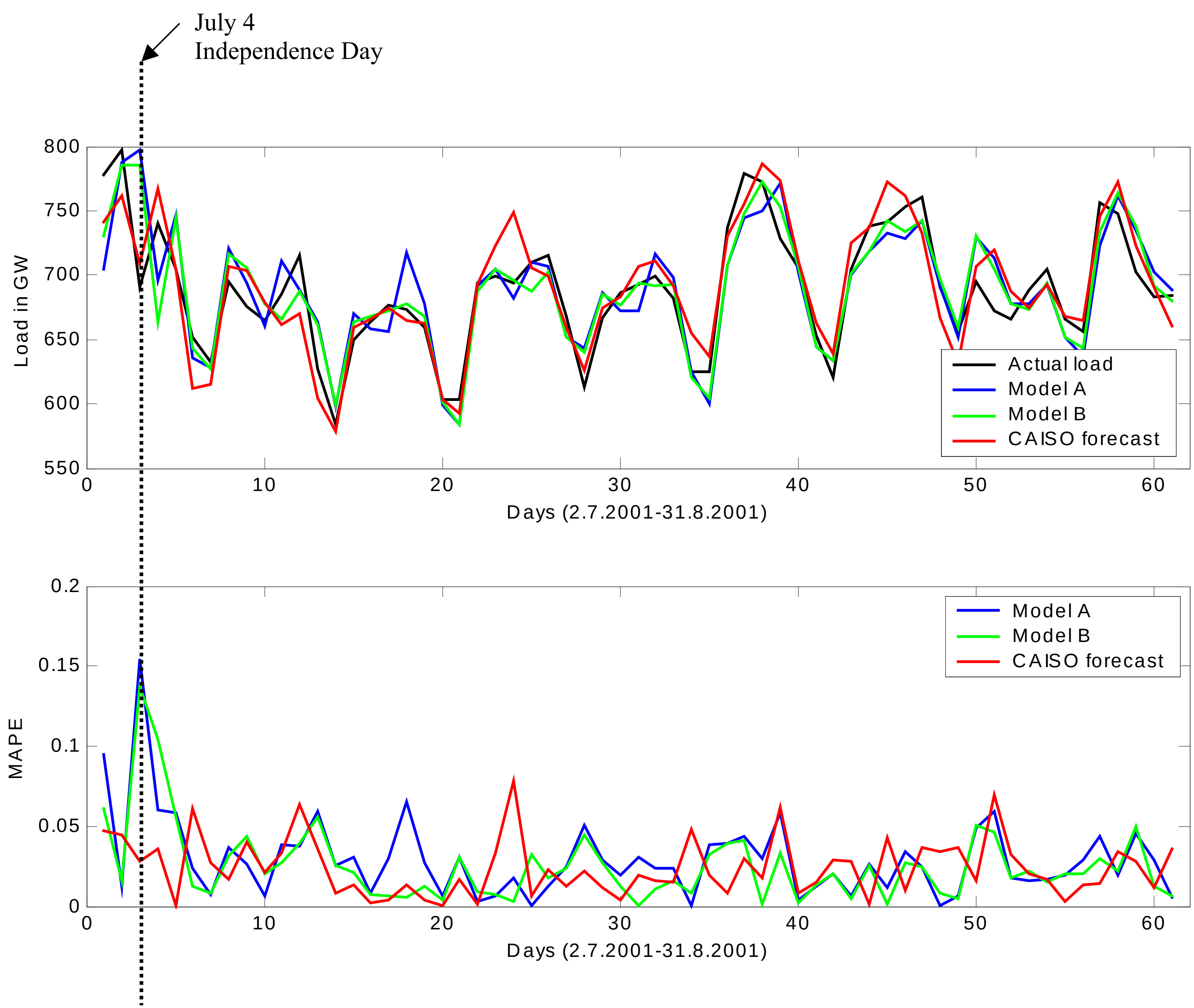
650
600
550
500
450
Load in GW
0
10
20
30
40
50
60
Days (3.3.2001-1.5.2001)
Actual load
Model A
Model B
CAISO forecast

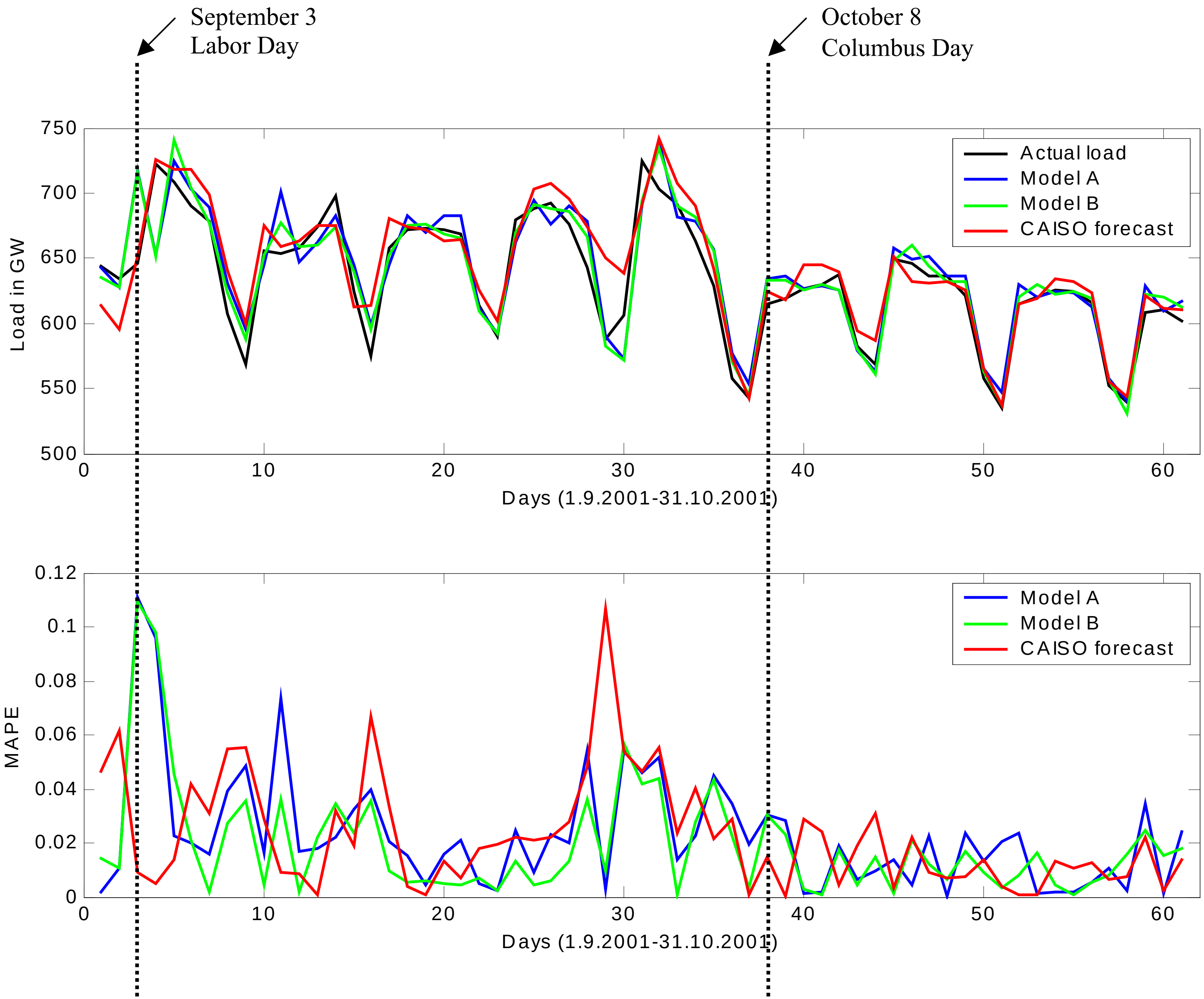


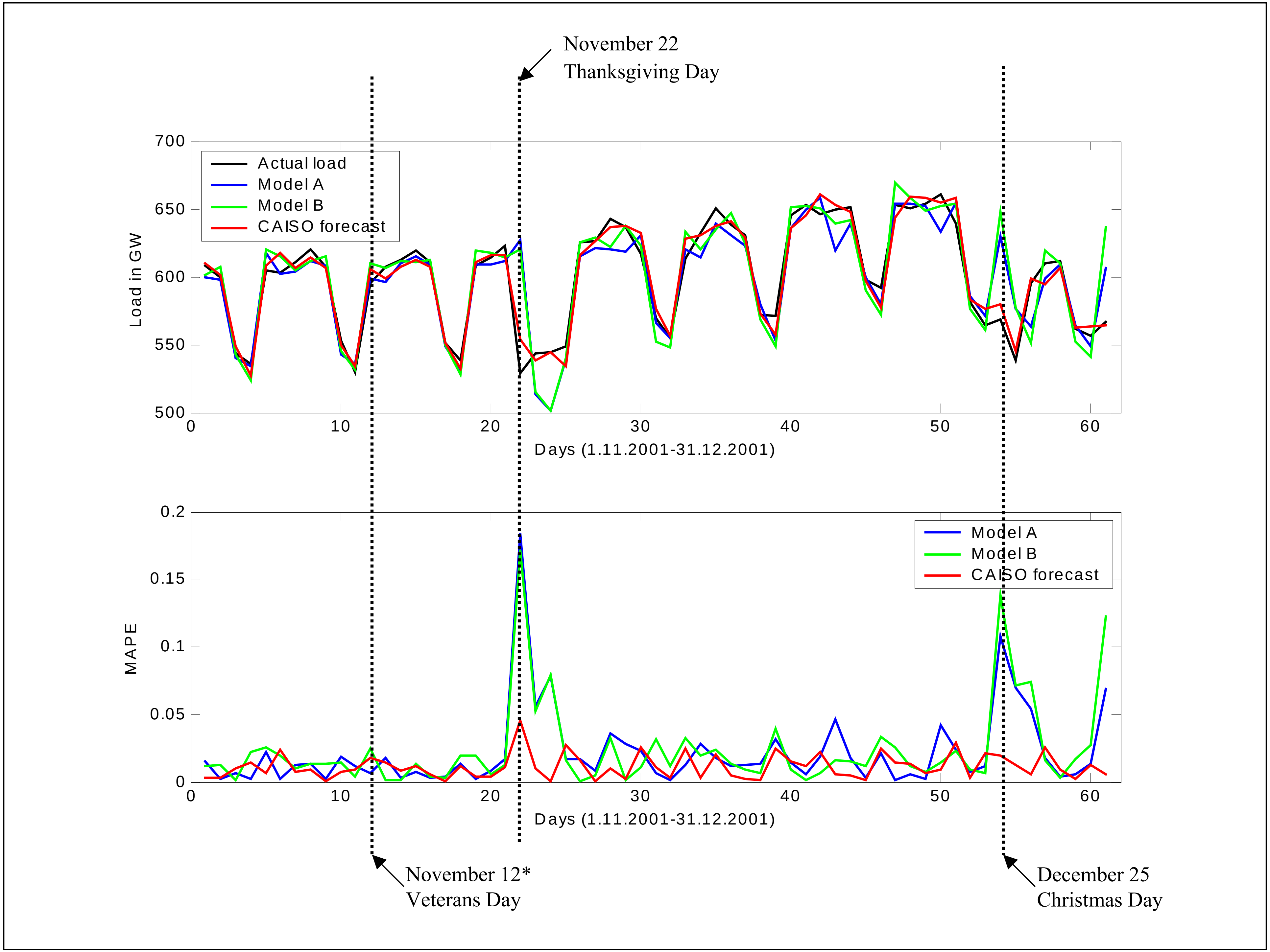
0.05
0.04
0.03
0.02
0.01
0
MAPE
0
10
20
30
40
50
60
Days (3.3.2001-1.5.2001)
Model A
Model B
CAISO forecast



May 28
Memorial Day
Load in GW
800
750
700
650
600
550
500
Actual load
Model A
Model B
CAISO forecast
0
10
20
30
40
50
60
Days (2.5.2001-1.7.2001)
MAPE
0.2
0.15
0.1
0.05
0
Model A
Model B
CAISO forecast
Days (2.5.2001-1.7.2001)


July 4
Independence Day
Load in GW
Days (2.7.2001-31.8.2001)
Actual load
Model A
Model B
CAISO forecast
MAPE
Days (2.7.2001-31.8.2001)
Model A
Model B
CAISO forecast


September 3
Labor Day
October 8
Columbus Day
Load in GW
750
700
650
600
550
500
0
10
20
30
40
50
60
Days (1.9.2001-31.10.2001)
Actual load
Model A
Model B
CAISO forecast
MAPE
0.12
0.1
0.08
0.06
0.04
0.02
0
0
10
20
30
40
50
60
Days (1.9.2001-31.10.2001)
Model A
Model B
CAISO forecast



November 22
Thanksgiving Day
Load in GW
700
650
600
550
500
Actual load
Model A
Model B
CAISO forecast
0
10
20
30
40
50
60
Days (1.11.2001-31.12.2001)
MAPE
0.2
0.15
0.1
0.05
0
Model A
Model B
CAISO forecast
Days (1.11.2001-31.12.2001)
November 12*
Veterans Day
December 25
Christmas Day

Forecast errors in 2001

MAPE (Jan.-Dec.)	CAISO	Model A	Model B
With holidays	1.84%	2.30%	2.08%
Without holidays	**1.77%**	**1.95%**	**1.71%**