Pricing derivatives in electricity markets

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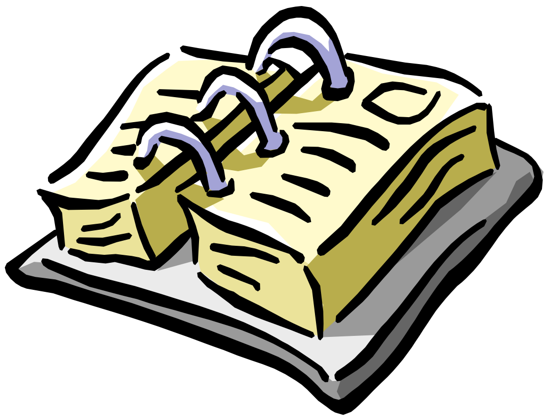
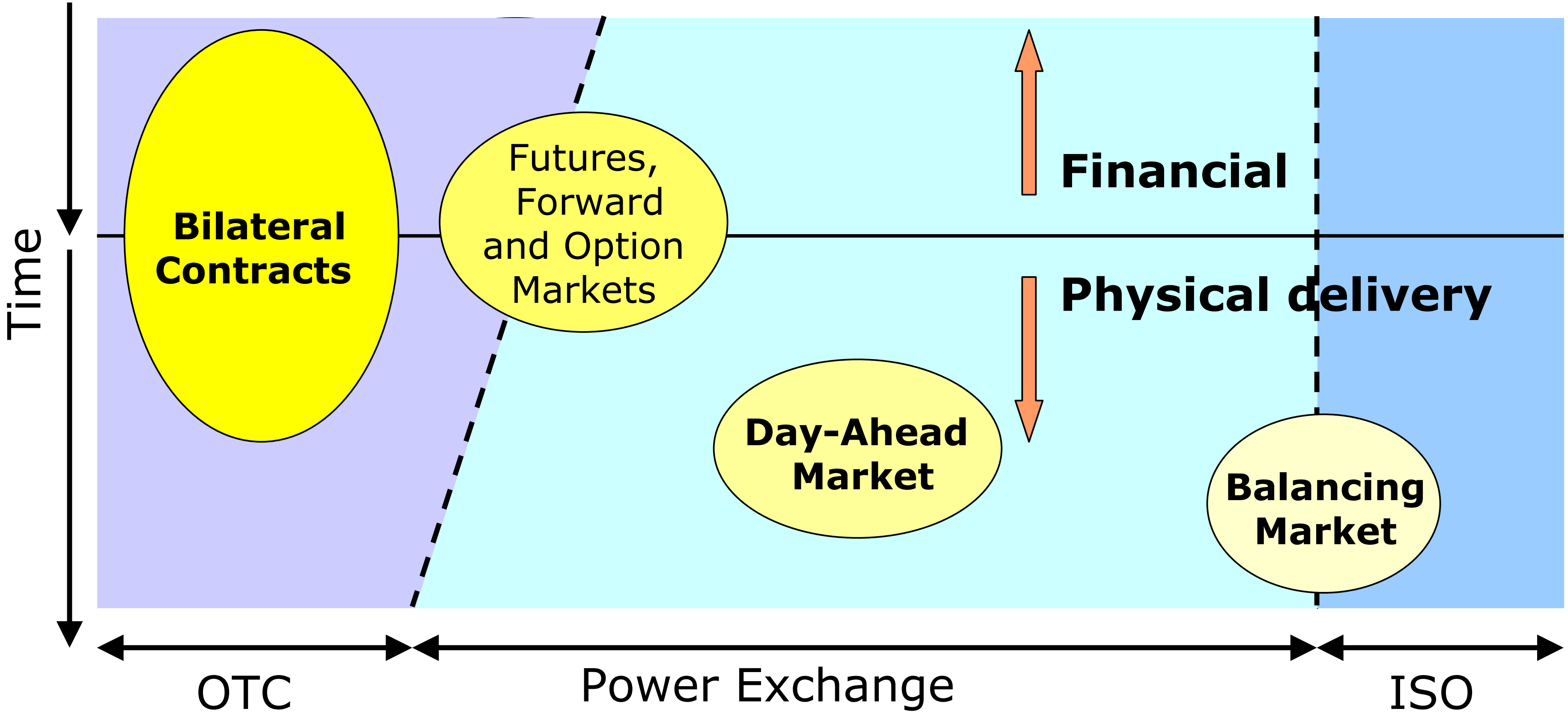
Agenda

- **Power markets in a nutshell**
- Modeling approaches
- Market price of risk implied by Asian options



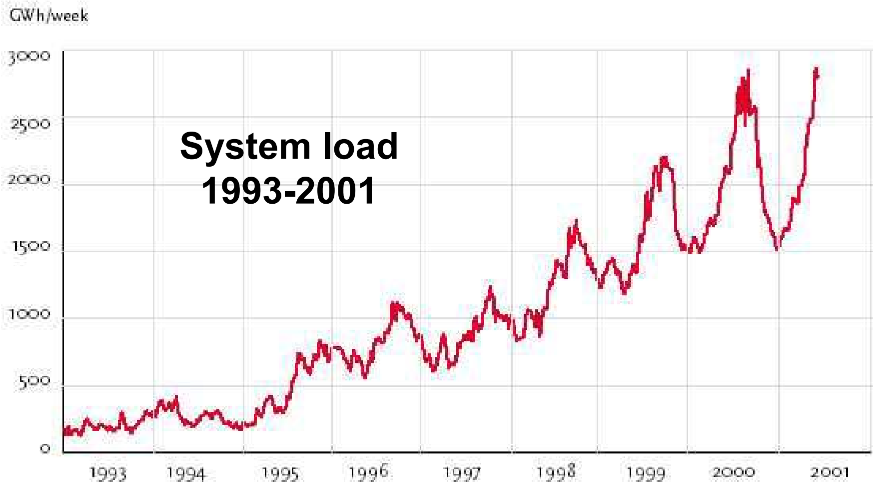
Wholesale electricity market structure

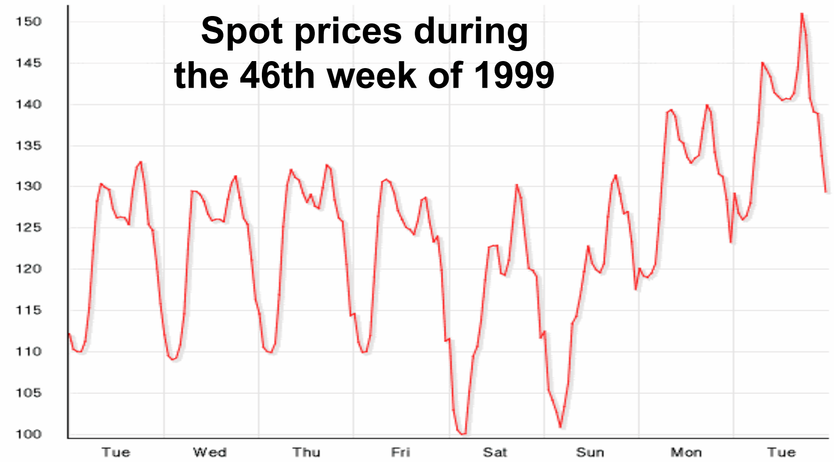


Unique character of electricity

- Seasonality
- Weather dependency
- Spikes in prices and loads (consumption)
  - Extreme volatility
- Non-storability or limited storability
  - Arbitrage pricing ?!

Load and price seasonality





NORD POOL
THE NORDIC POWER EXCHANGE

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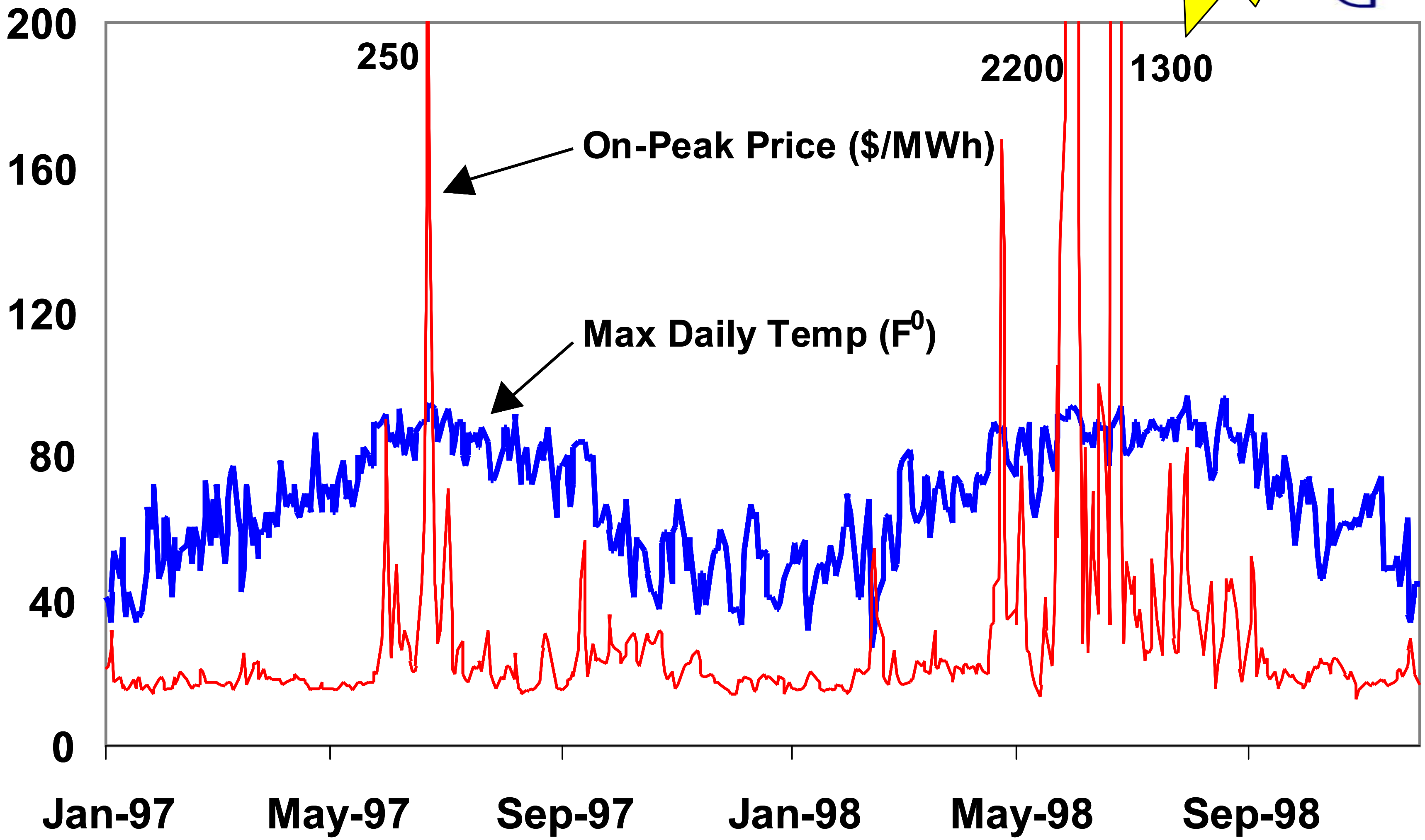
Cinergy – power price vs. maximum temperature



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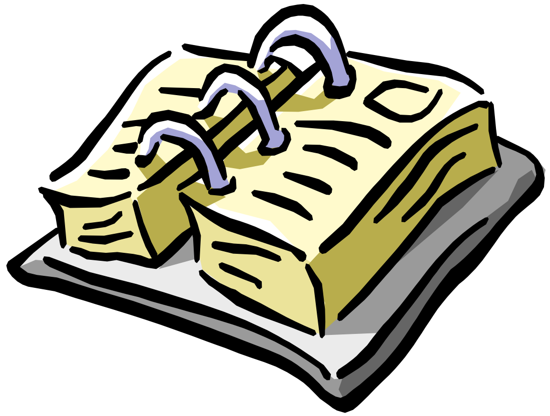
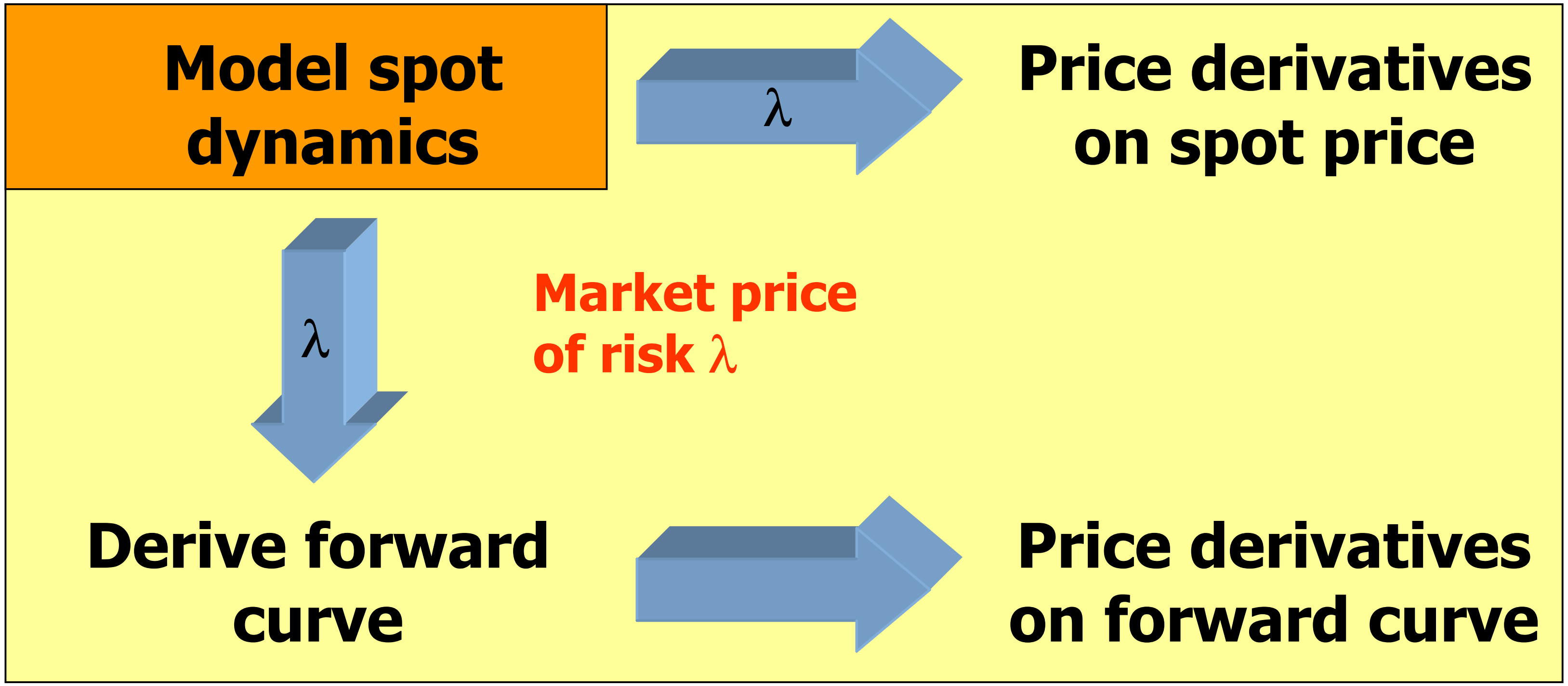


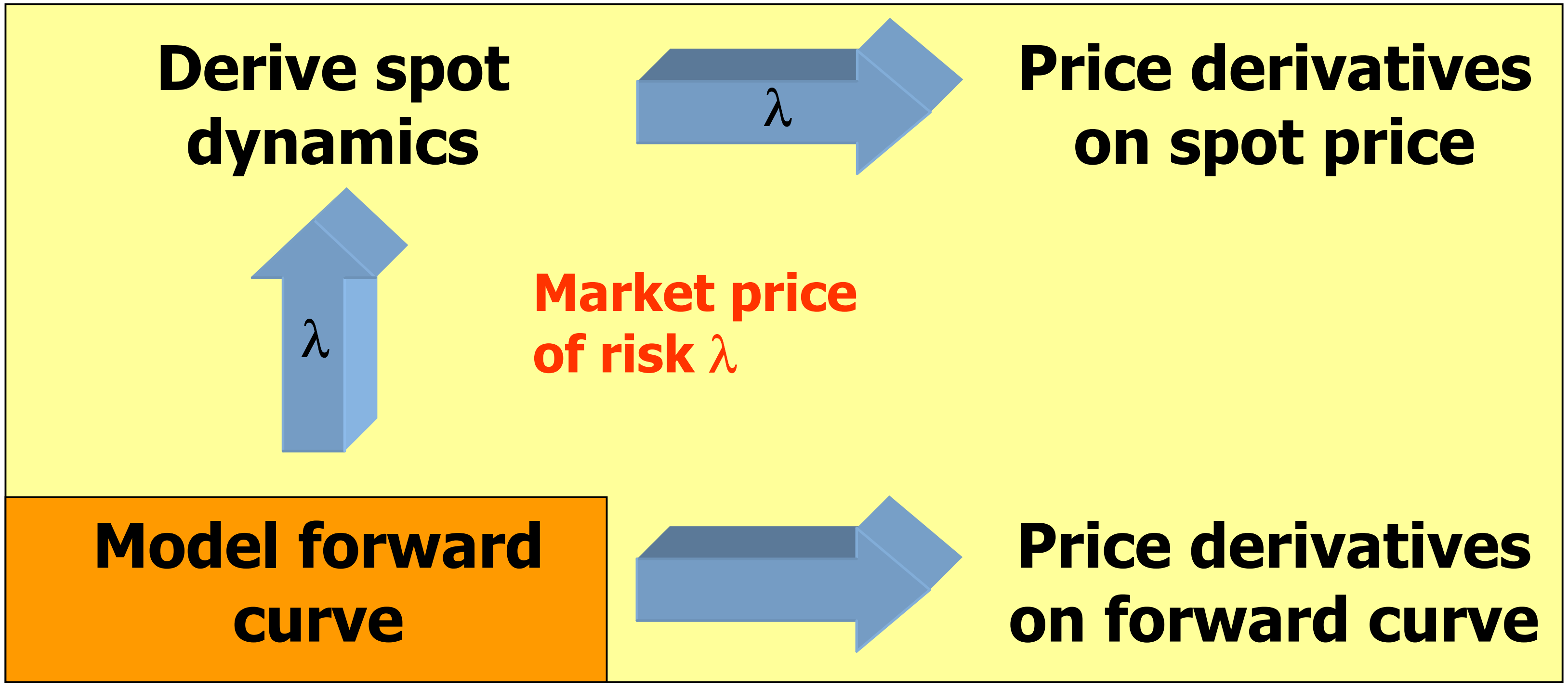
Modeling the spot price

- Econometric (also called: statistical) models
  - follow the finance tradition of modeling the stochastic processes that describe price properties
- Fundamental models
  - build the price processes based on equilibrium models for the electricity market

Derivative pricing



Derivative pricing cont.



Econometric models for the spot price

- Typically the spot electricity price is assumed to follow some kind of a jump-diffusion process:

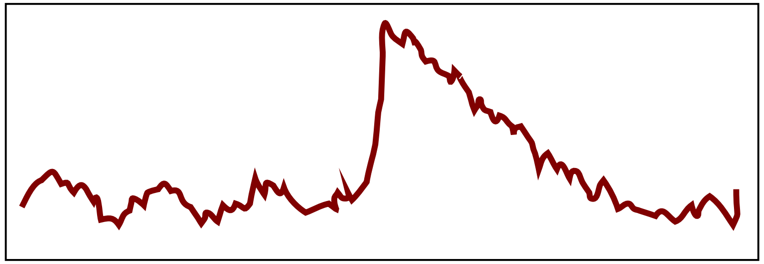
$$dS = \mu(S,t)dt + \sigma(S,t)dB + dq(S,t)$$

- where q(S,t) is a pure jump process with given intensity and severity (a compound Poisson process)

- Clewlow-Strickland, 2000; Eydeland-Geman, 2000; Johnson-Barz, 1999

Econometric models for the spot price cont.

- After a jump the price is forced back to its normal level by
  - mean reversion or
  - mean reversion coupled with downward jumps



- Alternatively, a positive jump may be always followed by a negative jump of (approximately) the same size – especially on the daily time scale

(Weron-Simonsen-Wilman, 2004)

Econometric models for the spot price cont.

- Burger-Klar-Müller-Schindlmayr (2004) incorporate a SARIMA forecast of the system load into the spot price

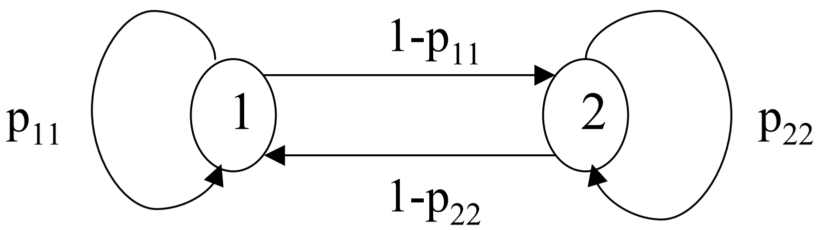
$$S_t = \exp(f(t, L_t / v_t) + X_t + Y_t)$$

- L_t: deterministic load forecast + SARIMA
- X_t: short term market fluctuations (SARIMA)
- $f(t, \cdot / v_t)$: deterministic function; specifies the expected relative availability of power plants
- Y_t: Brownian motion with drift; responsible for the variation of futures prices

Econometric models for the spot price cont.

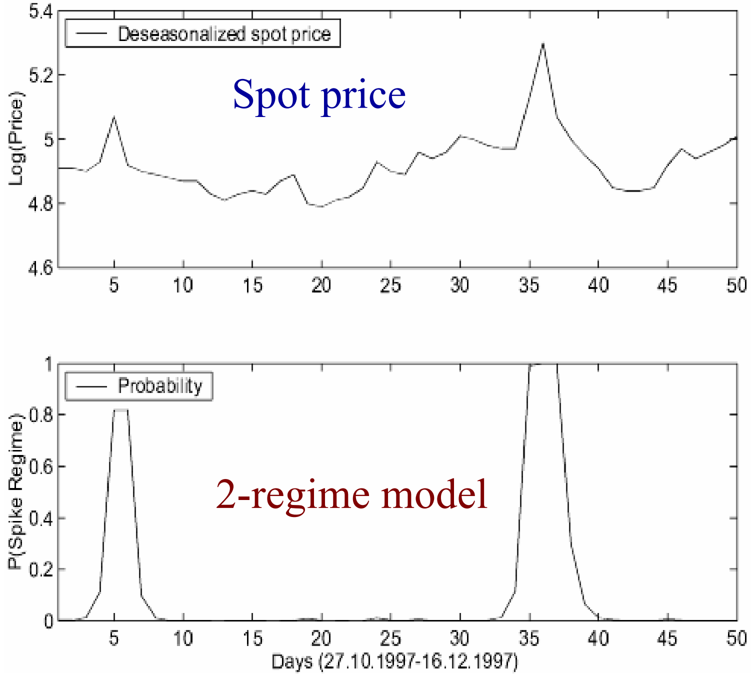
- Huisman-Mahieu (2003) assume that the switching mechanism can be governed by a random variable that follows a Markov chain with two possible states

A two-state regime model: $X_t = \{1,2\}$



Transition probabilities:

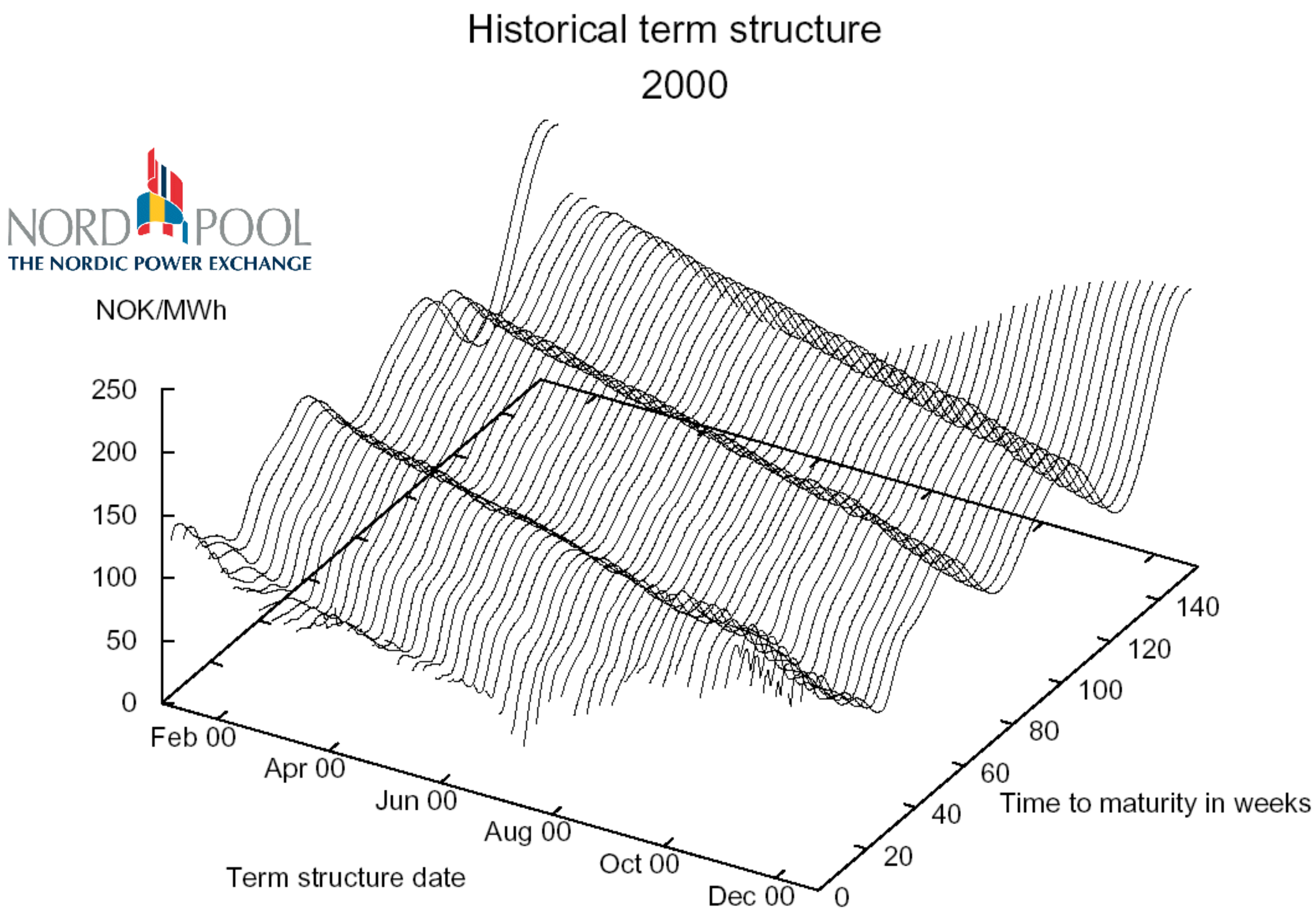
$$P = (p_{ij}) = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{pmatrix}$$



Fundamental models

- Skantze-Ilic (2001) considered a fundamental model that incorporated the seasonality of prices, stochastic supply outages, and mean reversion
- Barlow (2002) used a mean-reverting process for the demand and a fixed supply function to end up with a mean-reverting process for the spot price
- Based on stochastic climate factors (temperature, precipitation), Vahviläinen-Pyykkönen (2004) modeled hydrological inflow and snow-pack development that affect hydro power generation

Forward price dynamics models



Forward price dynamics models cont.

- Koekebakker-Ollmar (2001) use PCA to analyze the volatility factor structure of the forward curve
  - Conclusion: two stochastic factors are unable to explain the forward curve dynamics as well as in interest rate market
- Audet-Heiskanen-Keppo-Vehviläinen (2004) consider modeling the electricity forward curve dynamics with parameterized volatility and correlation structures

$$d f(t, T) = f(t, T) e^{-\alpha (T-t)} \sigma(T)\, dB_T(t)$$
$$dB_S(t)\, dB_T(t) = e^{-\rho |T-S|} dt$$

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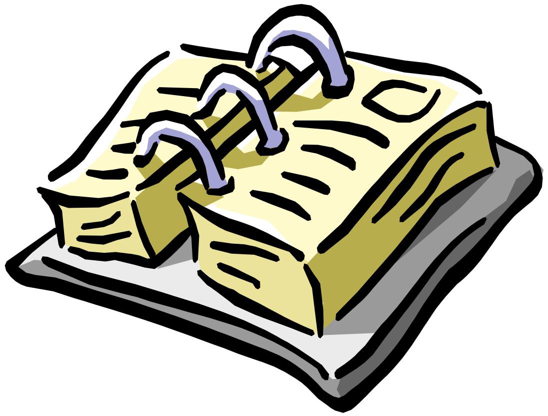
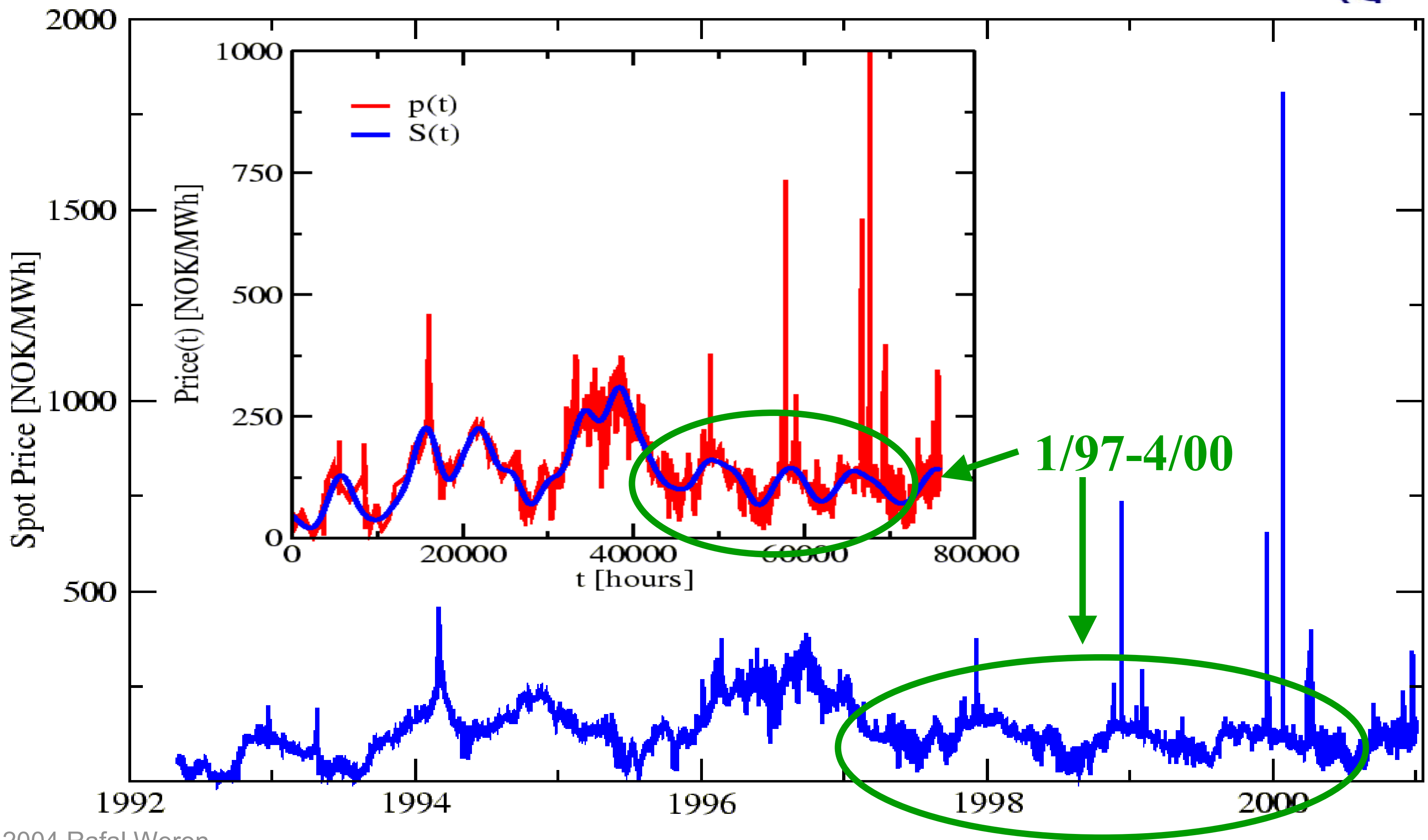
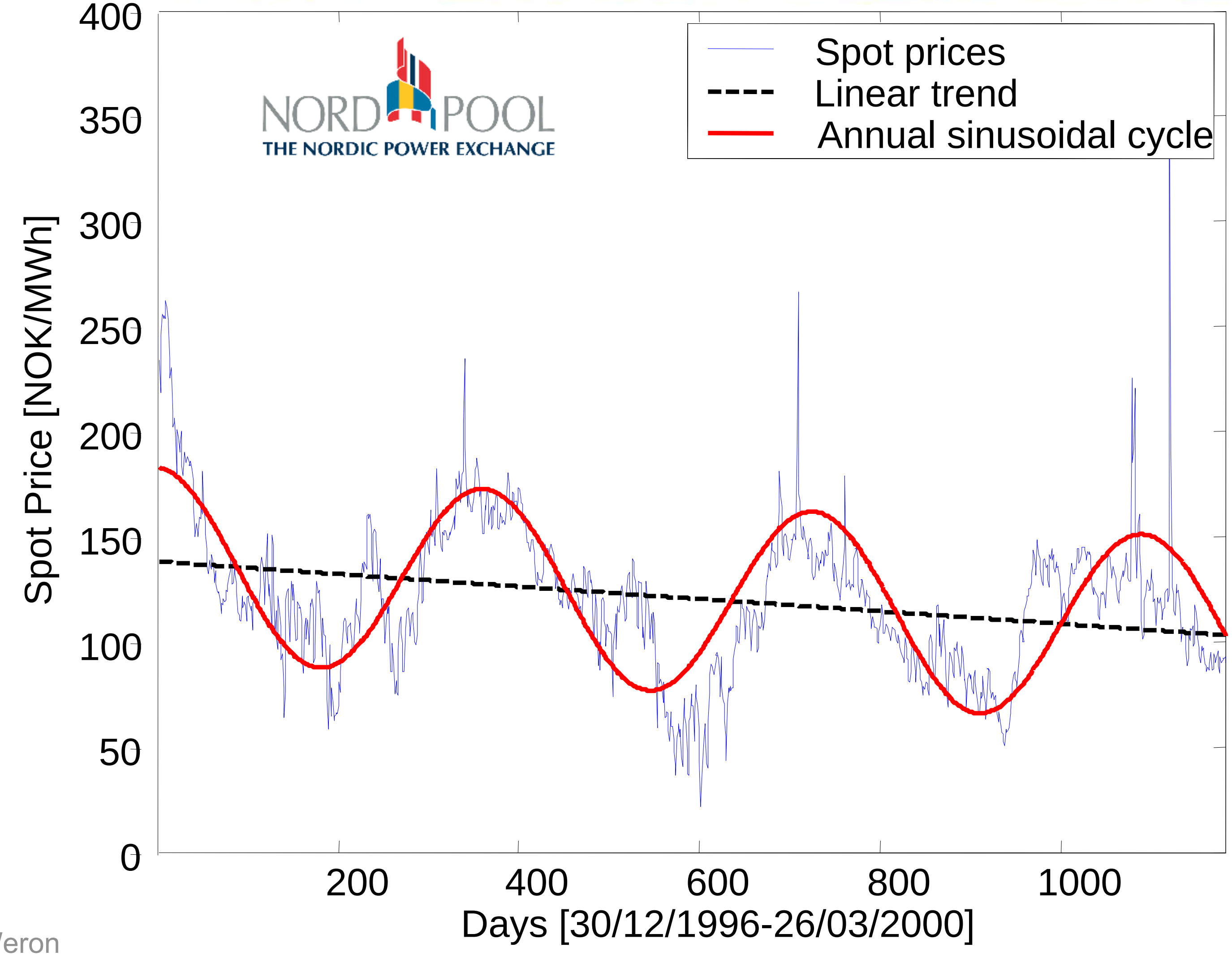


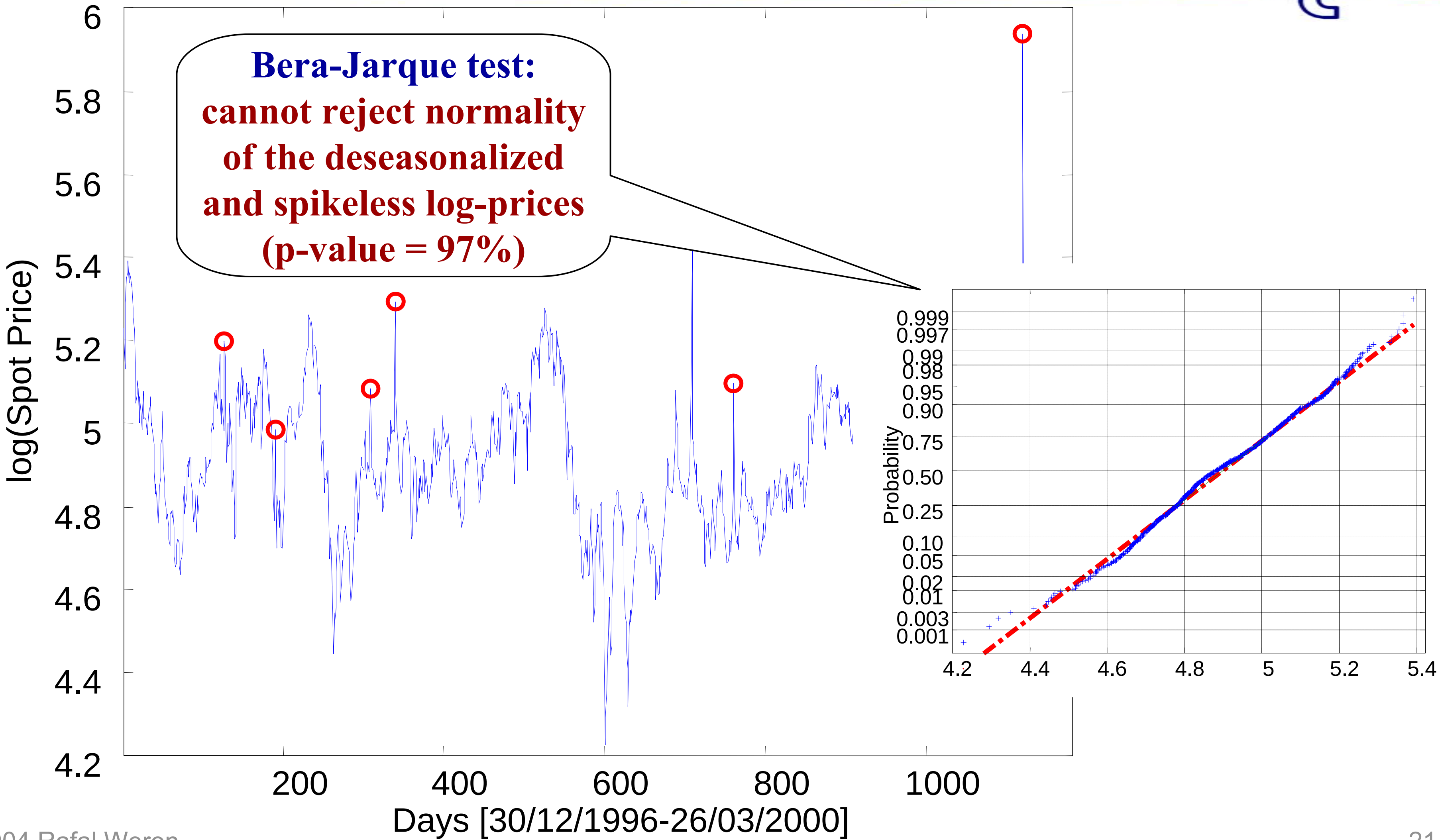
Nord Pool hourly spot prices: 1992-2000



Spot prices, linear trend and annual sinusoidal cycle



Deseasonalized log-spot prices



The Vasicek model

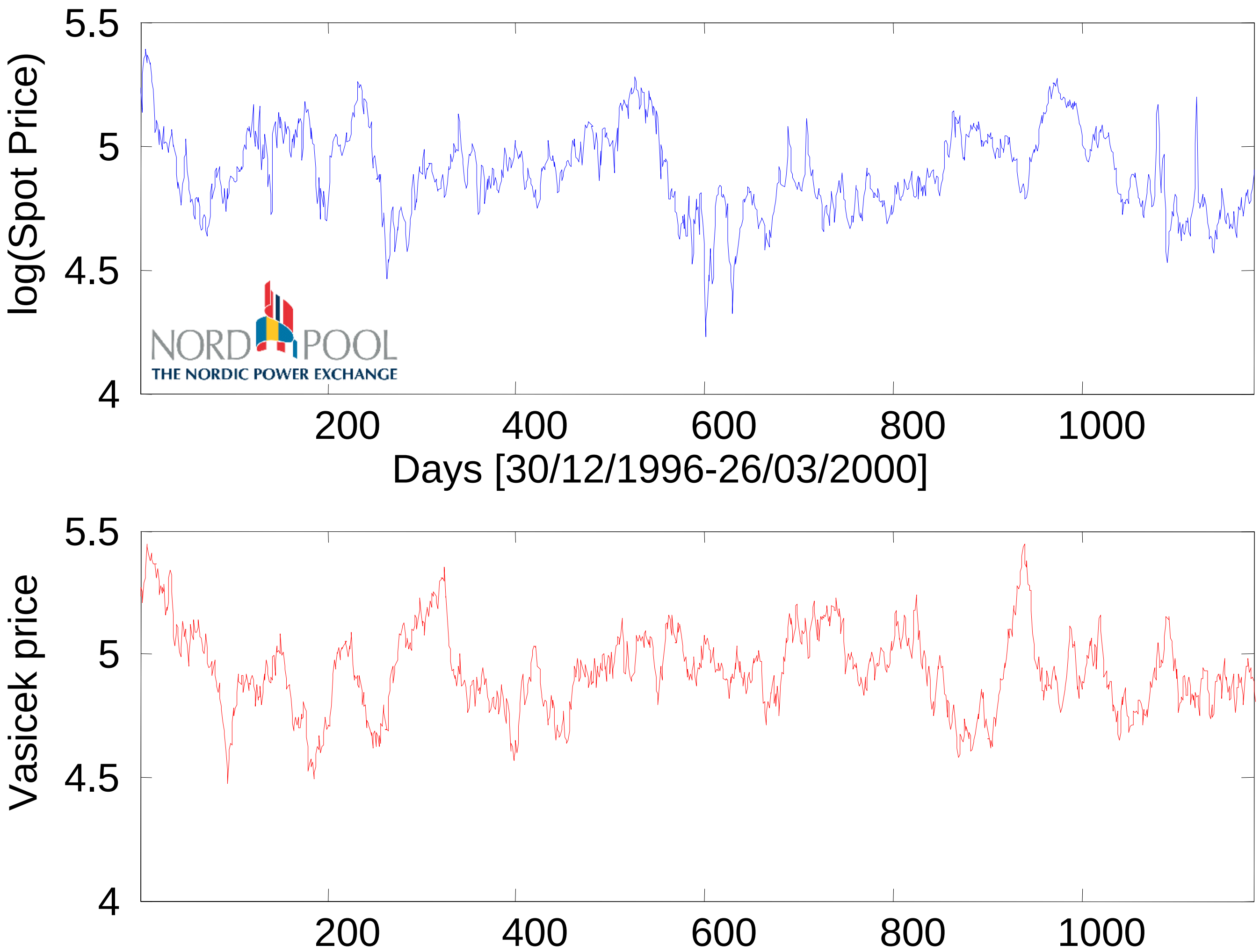
- Originally developed for spot rate modeling (1977)
- Defined as a generalized Ornstein-Uhlenbeck process

$$dX = (\alpha - \beta X)dt + \sigma dB = \beta\left(\frac{\alpha}{\beta} - X\right)dt + \sigma dB$$

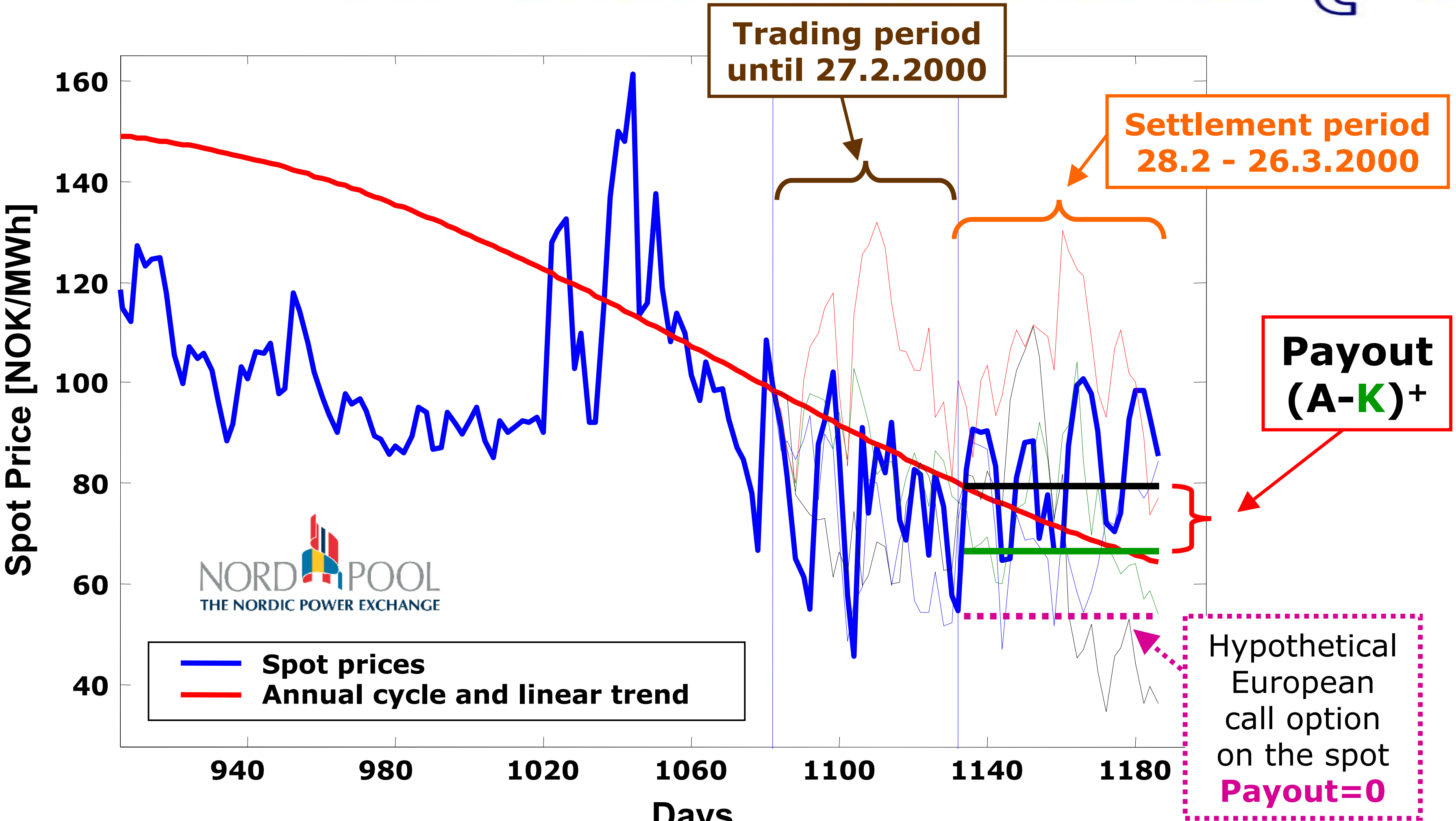
- Conditional distribution of X at time t is normal with

$$E(X_t) = \frac{\alpha}{\beta} + \left(X_0 - \frac{\alpha}{\beta}\right)e^{-\beta t} \qquad Var(X_t) = \frac{\sigma^2}{2\beta}\left(1 - e^{-2\beta t}\right)$$

Deseasonalized, spikeless spot prices and Vasicek simulation



Spot and simulated (Monte Carlo) prices

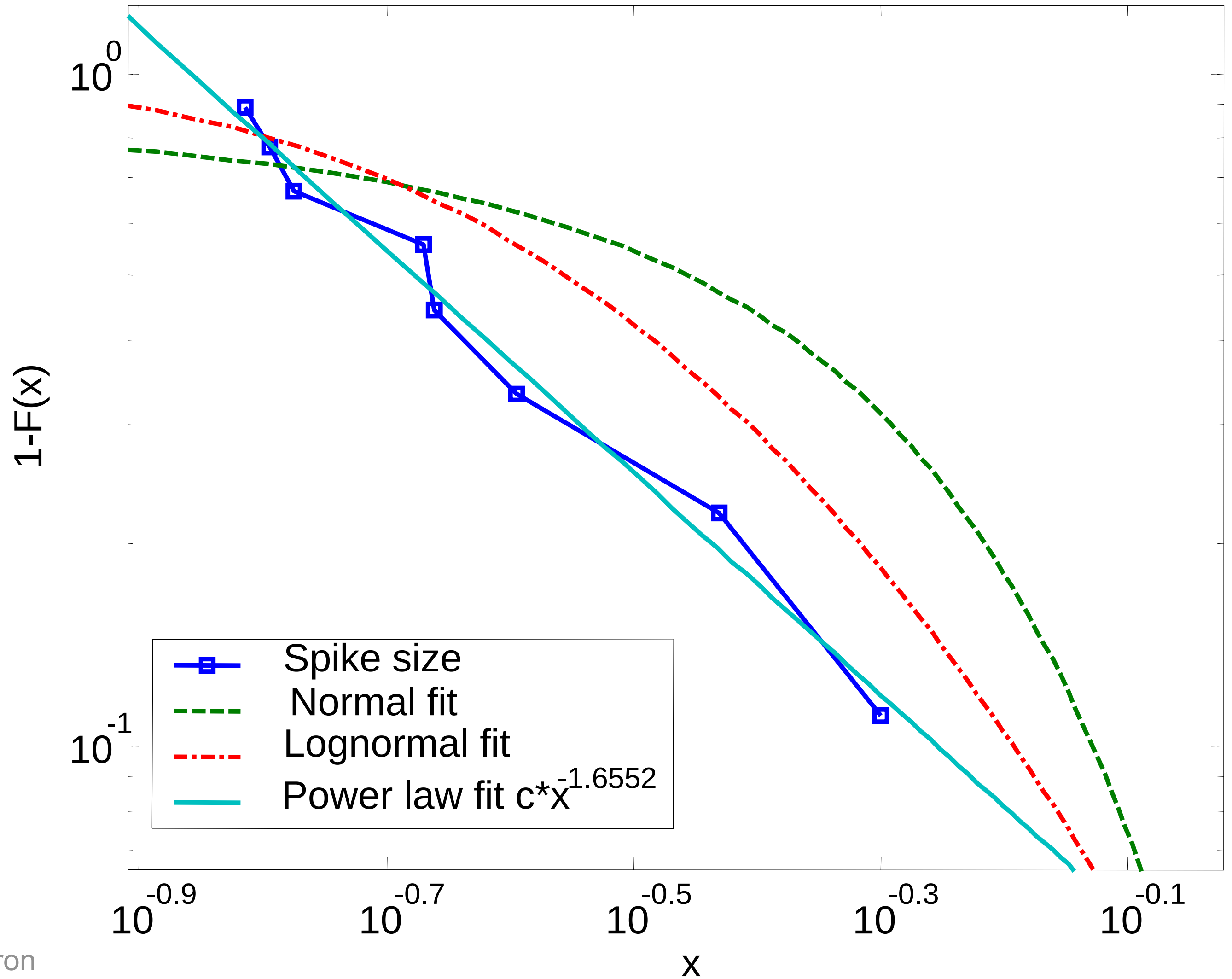


A model for spot prices

- We model the spot price as $\mathbf{p_t = s_t + S_t + exp(J_t dq_t + X_t)}$:
  - $\mathbf{s_t}$ is the weekly seasonal component (i.e. an average week),
  - $\mathbf{S_t = A \sin(2\pi(t + B)/365) + Ct}$ is the annual seasonal component,
  - $\mathbf{J_t\, dq_t}$ is the jump component: $\mathbf{q_t}$ is a Poisson r.v. with intensity κ and $\mathbf{J_t}$ is a random jump size, e.g. a lognormal r.v. log $\mathbf{J_t} \sim N(\mu,\rho^2)$ truncated at the maximum price attainable in the market
  - $\mathbf{X_t}$ is a generalized Ornstein-Uhlenbeck type process:

$$dX_t = (\alpha - \beta X_t)dt + \sigma dB_t = \beta(\tfrac{\alpha}{\beta} - X_t)dt + \sigma dB_t$$

Spike size distribution is heavy-tailed



Market price of risk (λ)

- Financial theory (Cox-Ross, 1976; Harrison-Kreps, 1979) says: **to price derivatives we should simulate prices according to the risk-neutral (or risk-adjusted) process**
- In math-finance language: we have to change the measure

$$
\begin{aligned}
dX_t &= (\alpha - \beta X_t)dt + \sigma dB_t = \\
&= (\alpha - \beta X_t)dt + \sigma d(B_t^\lambda - \lambda t) = \\
&= \beta\left(\frac{\alpha - \lambda\sigma}{\beta} - X_t\right)dt + \sigma dB_t^\lambda
\end{aligned}
$$

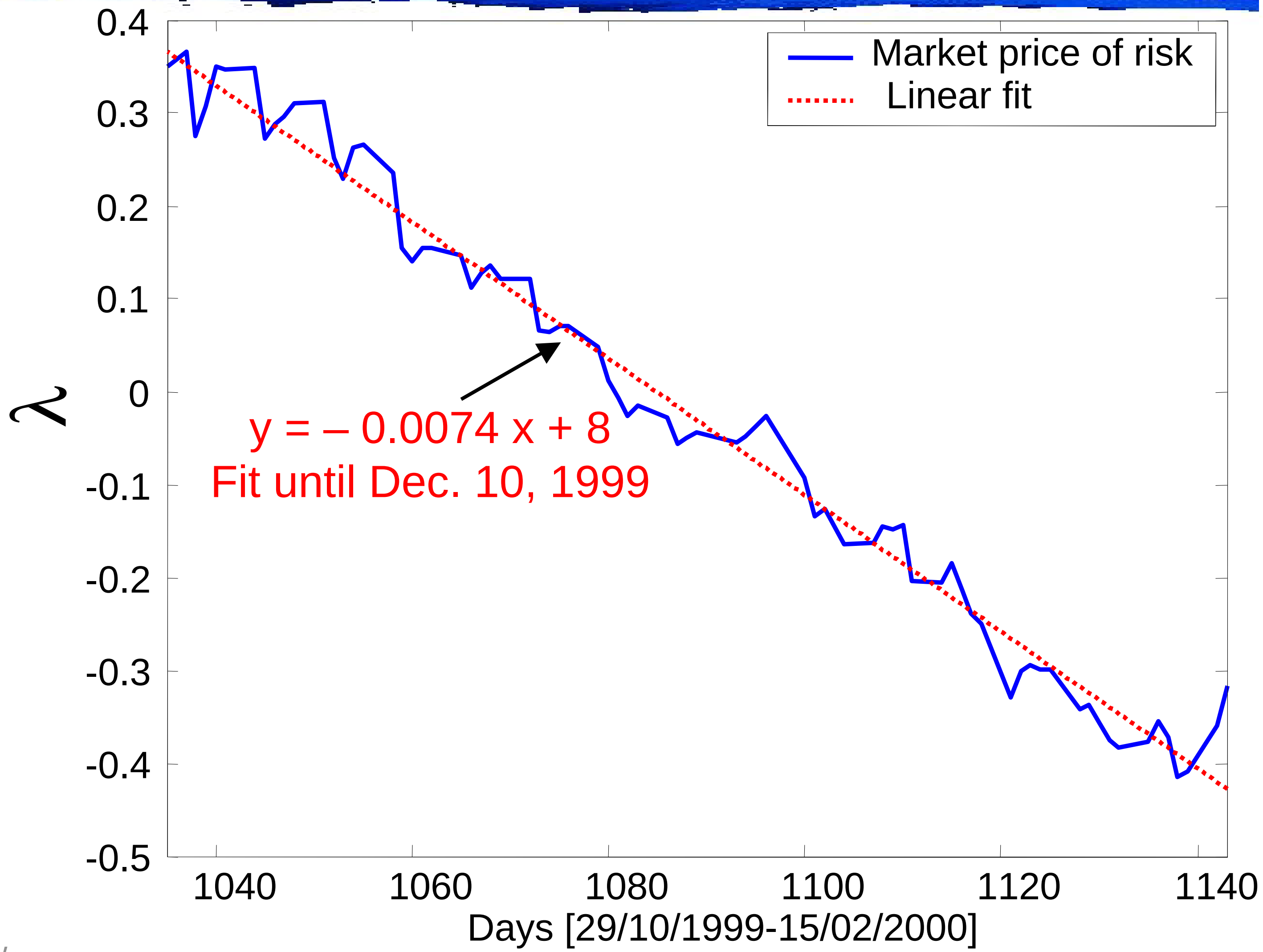
- Now the process has a new level of mean reversion
- We can find λ by calibrating the model to option prices

Market price of risk (λ) cont.

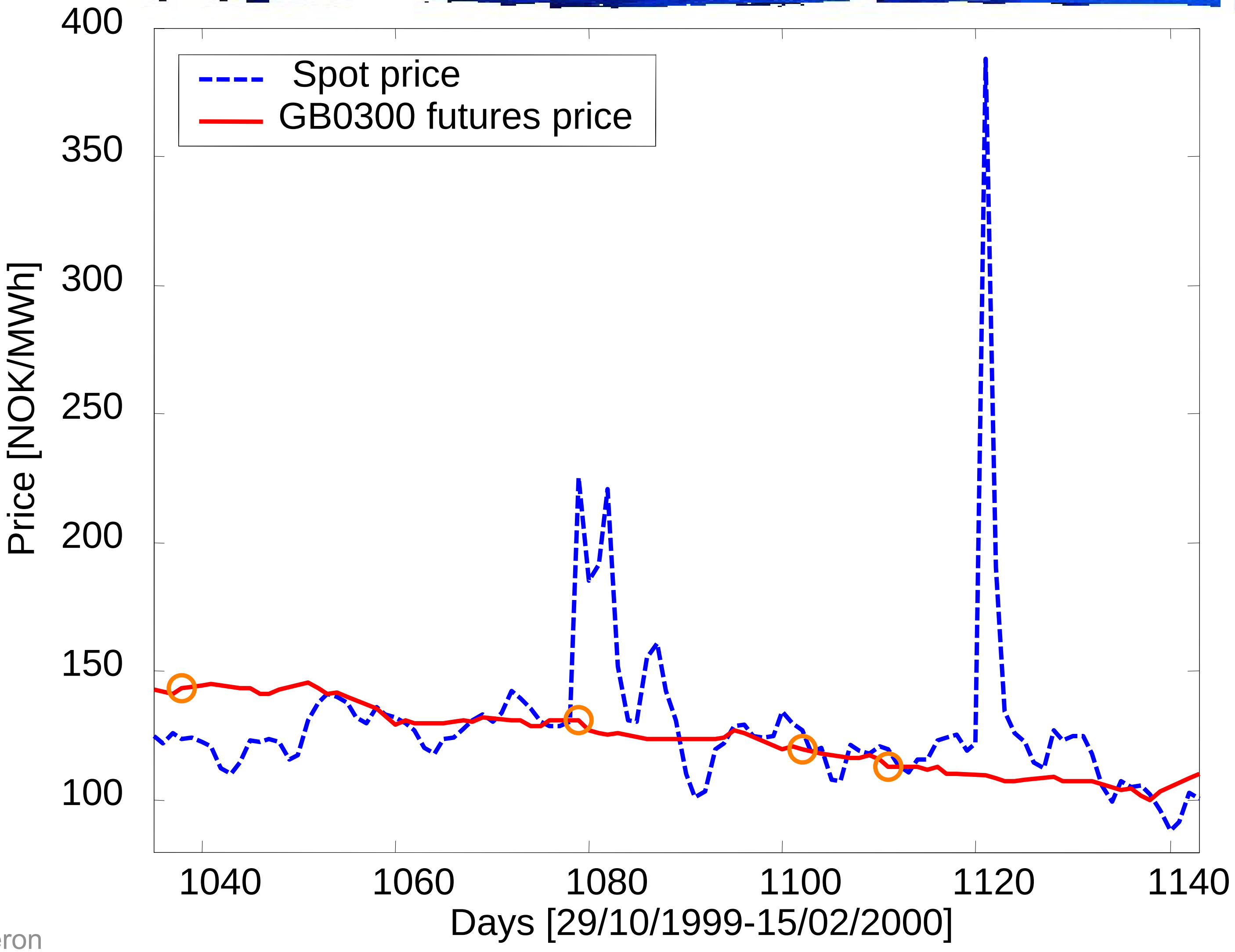
- For spot electricity prices until 26.03.2000 the GMM calibration (*O-U part*), MLE (*"spiky" part*) and OLE (annual sinusoidal cycle) yielded the following estimates

  - $\alpha = 0.2760$, $\beta = 0.0560$, $\sigma = 0.0459$ (*O-U part*)
  - $\kappa = 0.00762$, $\mu = -1.2774$, $\rho = 0.6512$ (*"spiky" part*)
  - $A = 45.19$, $B = 94.8$, $C = -0.0295$ (*annual sinusoidal cycle*)

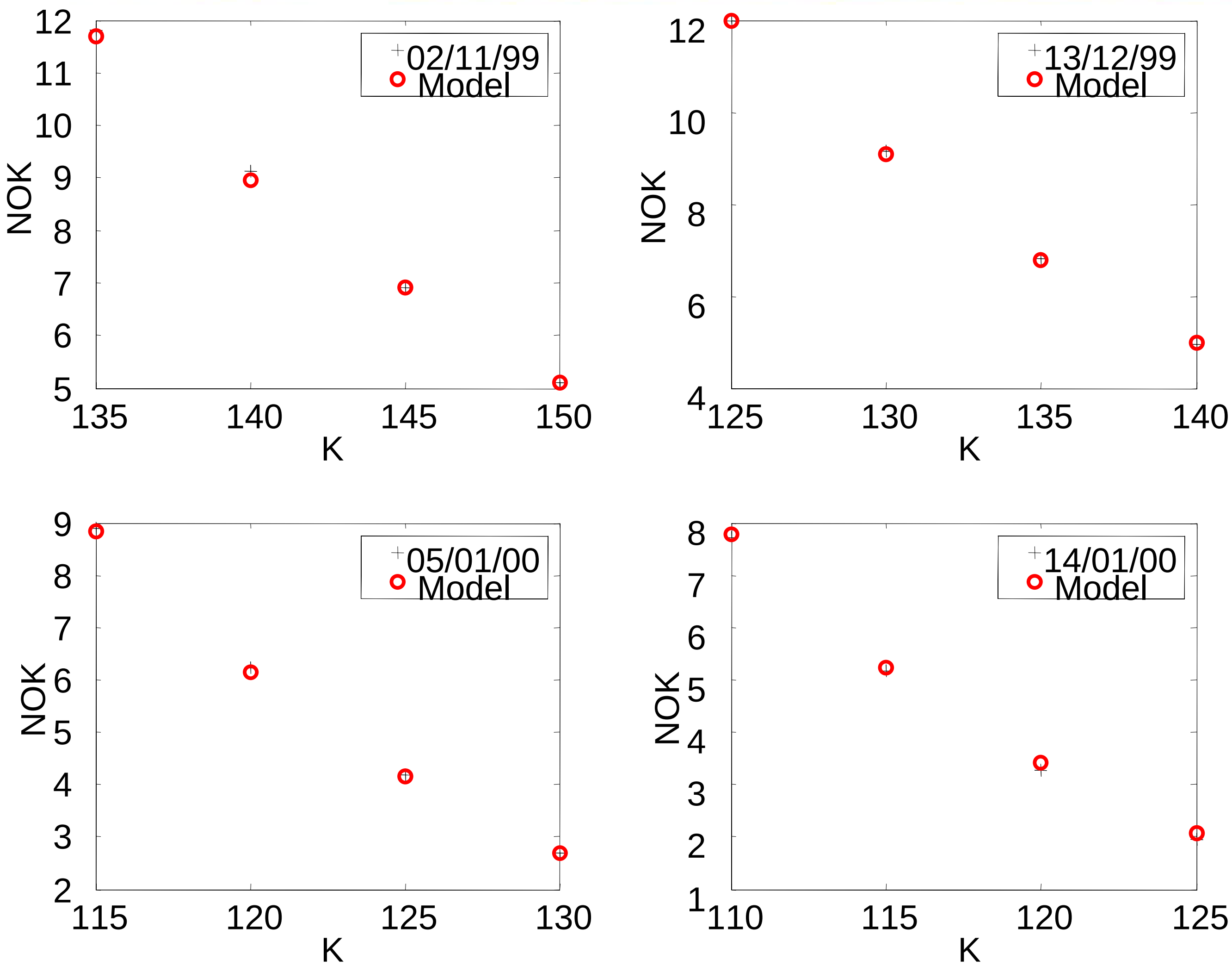
Market price of risk (λ) cont.



Spot and futures price



Asian call option prices



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