Heavy tails and electricity prices

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Agenda

- The world is changing …
- Why do we care?
- What can we do about it?
The world is changing …

- Power market liberalization
  - privatization
  - new actors
  - new markets
  - acquisitions by foreign investors
  - mergers

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>1990</td>
<td>England &amp; Wales Electricity Pool⁶</td>
</tr>
<tr>
<td>Norway</td>
<td>1992</td>
<td>Nord Pool⁵</td>
</tr>
<tr>
<td>Sweden</td>
<td>1996</td>
<td>Nord Pool</td>
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<tr>
<td>Spain</td>
<td>1998</td>
<td>Operadora del Mercado Español de Electricidad (OMEL)³</td>
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<tr>
<td>U.S.</td>
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<td>California Power Exchange (CalPX)⁴</td>
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<tr>
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<td>New York ISO (NYISO)</td>
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<tr>
<td>Germany</td>
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<td>Leipzig Power Exchange (LPX)⁵</td>
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<tr>
<td>Germany</td>
<td>2000</td>
<td>European Energy Exchange (EEX)</td>
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<tr>
<td>Denmark</td>
<td>2000</td>
<td>Nord Pool</td>
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<tr>
<td>Poland</td>
<td>2000</td>
<td>Towarowa Gieda Energii (Polish Power Exchange, PolPX)</td>
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<tr>
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<td>Pennsylvania-New Jersey-Maryland (PJM) Interconnection</td>
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<tr>
<td>U.K.</td>
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<td>UK Power Exchange (UKPX)⁷</td>
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<tr>
<td>U.K.</td>
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<td>Automated Power Exchange (APX UK)⁸</td>
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<tr>
<td>Slovenia</td>
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<td>Borzen</td>
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<tr>
<td>Poland</td>
<td>2002</td>
<td>Platforma Obrotu Energii Elektrycznâ (POEE)</td>
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<tr>
<td>France</td>
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<td>Powernext</td>
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<tr>
<td>Austria</td>
<td>2002</td>
<td>Energy Exchange Austria (EXAA)</td>
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<tr>
<td>U.S.</td>
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<td>ISO New England</td>
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<tr>
<td>Italy</td>
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<td>Italian Power Exchange (IPEX)</td>
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<tr>
<td>Czech Rep.</td>
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<td>Operátor Trhu s Elektrinou (OTE)</td>
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<td>U.S.</td>
<td>2005</td>
<td>Midwest ISO (MISO)</td>
</tr>
<tr>
<td>Belgium</td>
<td>2006</td>
<td>Belgian Power Exchange (Belpex)</td>
</tr>
</tbody>
</table>
The world is changing ...

- Power market liberalization
  - privatization
  - new actors
  - new markets
  - acquisitions by foreign investors
  - mergers

- UK Pool, UK PX, APX, IPE (1990, …)
- Powernext (2002)
- APX (1999)
- Nord Pool (1992)
- TGE (PolPX) (2000)
- EXAA (2002)
- EEX/LPX (2000)
- IPEX (2004)
- Omel (1998)
- Borzen (2001)
Why deregulate?

- Belief that the success of liberalization in other industries can be duplicated in the power sector.
- Political “need” for splitting the vertically integrated monopoly structures.
Agenda

- The world is changing …
- Why do we care?
- What can we do about it?
Our bills
Blackouts
Wheel of misfortune

- General fraud (1991)
  - Speculation by Nick Leeson (1995)
  - Interest rate speculation (1998)
  - Questionable trades (1996)

- Bad investments & creative accounting (2001)
  - Retail price cap & wholesale price spike (2001)
  - Oil forward-futures “hedge” (1993)
  - Counterparty defaults (1998)
  - Accounting fraud (1998)

- Real estate losses (1994)
  - Asbestos cases settlement (1995)

- Banks
  - $1.1 bln
  - $3.8 bln
  - $1.3 bln
  - $0.3 bln
  - $1.3 bln

- Asset management
  - $10 bln
  - $0.24 bln
  - $0.3 bln

- Energy
  - $40 bln
  - $5.3 bln
  - $1 bln

- Insurance
  - Confed Life
  - Lloyd's

- Machinery
  - $8.9 bln

- PCA
  - $1 bln

- Energy Co
  - $1.1 bln

- Enron
  - $10 bln

- Morgan Grenfell
  - $1.3 bln

- BCCI
  - $0.3 bln

- PG&E
  - $1 bln

- Cendant
  - $0.24 bln

- Barings
  - $1.1 bln

- Daiwa
  - $3.8 bln

- LTCM
  - $0.3 bln

- Real estate losses (1994)
- Asbestos cases settlement (1995)
- Speculation by Nick Leeson (1995)
- Interest rate speculation (1998)
- Questionable trades (1996)
- General fraud (1991)
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- Oil forward-futures “hedge” (1993)
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Agenda

- The world is changing …
- Why do we care?
- What can we do about it?
What can we do about it?

- Modeling and forecasting
- Long-term
  - Profitability analysis and planning
- Medium-term (daily prices, monthly horizons)
  - Risk management, derivatives pricing
- Short-term (hourly prices, daily horizons)
  - Spot (24 hourly auctions) bidding
Unique character of electricity

- Seasonality
- Weather dependency
- Price spikes
- Extreme volatility
Load (consumption) and price seasonality

Spot prices during the 46th week of 1999

Average daily spot prices
System load vs. temperatures

Average daily load

Average daily temperature

Days (1.1.1999-30.9.2000)
Cinergy – power price vs. maximum temperature

Max Daily Temp (F$^0$)

On-Peak Price ($/MWh)

1997-1998
Price spikes
(at hourly resolution)
Price formation: power pool vs. exchange

Power pool: one-sided auction

Power exchange: two-sided auction
Price formation cont.: the supply stack

- Coal
- Oil
- Gas
- Nuclear
- Hydro
- Total production volume
- Marginal production cost

Suppliers stack
- Low demand curve
- High demand curve
Price formation cont.: the supply stack

- Supply stack
- Demand curve
- Low supply stack

Total production volume vs. Marginal production cost

- Nuclear
- Hydro
- Coal
- Oil
- Gas
There are markets with no price spikes
Extreme volatility:
bonds < 0.5%, stocks < 4%, oil/NG < 4% (daily scale)
Mean reversion

- Mean/floor reversion
  - Hurst exponent significantly below 0.5
  - R/S, DFA, GPH yield $0.25 < H < 0.4$
  - Confirmed by wavelet statistics AWC

- Short term persistence
Negative (!) prices

Hour of the day (Nov. 29th, 1999)

$/MWh

CalPX prices
Palo Verde prices

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Distribution of EEX electricity price changes

(c) 2005 Rafal Weron
Distribution of deseasonalized EEX price changes

\begin{table}
\centering
\begin{tabular}{|l|c|c|c|c|}
\hline
Parameters & $\alpha$ & $\sigma, \delta$ & $\beta$ & $\mu$ \\
\hline
Gaussian fit & 11.4548 & & 0.0083 & \\
Hyperbolic fit & 0.2099 & 0.0851 & $-0.0001$ & 0.0136 \\
NIG fit & 0.0469 & 3.2181 & $-0.0031$ & 0.0083 \\
$\alpha$-stable fit & 1.5104 & 2.9005 & $-0.2616$ & $-0.4898$ \\
\hline
Test values & Anderson-Darling & Kolmogorov & \\
Gaussian fit & $+\text{INF}$ & 6.9894 & \\
Hyperbolic fit & $+\text{INF}$ & 1.8669 & \\
NIG fit & 1.7890 & 0.9138 & \\
$\alpha$-stable fit & 0.5419 & 0.6831 & \\
\hline
\end{tabular}
\end{table}
Distribution of deseasonalized EEX log-price changes

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(\alpha)</th>
<th>(\sigma, \delta)</th>
<th>(\beta)</th>
<th>(\mu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian fit</td>
<td>24.4395</td>
<td>0.0445</td>
<td></td>
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<tr>
<td>Hyperbolic</td>
<td>0.0664</td>
<td>0.3653</td>
<td>0.0001</td>
<td>0.0047</td>
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<td>NIG fit</td>
<td>0.0233</td>
<td>12.6781</td>
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<tr>
<td>(\alpha)-stable fit</td>
<td>1.4837</td>
<td>9.9668</td>
<td>-0.1915</td>
<td>-1.3267</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Test values</th>
<th>Anderson-Darling</th>
<th>Kolmogorov</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian fit</td>
<td>+INF</td>
<td>4.0124</td>
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<tr>
<td>Hyperbolic</td>
<td>2.6215</td>
<td>1.1440</td>
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<tr>
<td>NIG fit</td>
<td>0.7570</td>
<td>0.7752</td>
</tr>
<tr>
<td>(\alpha)-stable fit</td>
<td>0.5237</td>
<td>0.6603</td>
</tr>
</tbody>
</table>

(c) 2005 Rafal Weron
Distribution of deseasonalized Nord Pool log-price changes

Average daily spot prices

Spot Price [NOK/MWh]

Days [30/12/1996-26/03/2000]

Log(Spot Price)

Days [30/12/1996-26/03/2000]
Stable law yields a nearly perfect fit

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\alpha$</th>
<th>$\sigma, \delta$</th>
<th>$\beta$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian fit</td>
<td></td>
<td>7.0514</td>
<td></td>
<td>-0.0280</td>
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<tr>
<td>Hyperbolic fit</td>
<td>0.2587</td>
<td>0.8437</td>
<td>0.0029</td>
<td>-0.1179</td>
</tr>
<tr>
<td>NIG fit</td>
<td>0.1033</td>
<td>3.6808</td>
<td>-0.0009</td>
<td>-0.0280</td>
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<tr>
<td>$\alpha$-stable fit</td>
<td>1.6078</td>
<td>2.8062</td>
<td>-0.0115</td>
<td>-0.0431</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test values</th>
<th>Anderson-Darling</th>
<th>Kolmogorov</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian fit</td>
<td>+INF</td>
<td>4.3882</td>
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<tr>
<td>Hyperbolic fit</td>
<td>+INF</td>
<td>0.9933</td>
</tr>
<tr>
<td>NIG fit</td>
<td>0.9621</td>
<td>0.7333</td>
</tr>
<tr>
<td>$\alpha$-stable fit</td>
<td>0.2025</td>
<td>0.4861</td>
</tr>
</tbody>
</table>

The diagram shows the comparison of various distributions with the log (CDF(Return)) plotted against log(Return). The red line represents the $\alpha$-stable fit, which shows a nearly perfect fit compared to other distributions.
Modeling electricity spot prices

- Production-cost (or cost-based) models
- Equilibrium (or game theoretic) approaches
- Fundamental (or structural) methods
- **Quantitative** (or **stochastic**, **econometric**, **reduced-form**) models,
  - which characterize the statistical properties of electricity prices over time, with the ultimate objective of derivatives evaluation and risk management
- Statistical (or technical analysis) approaches
- AI-based (or non-parametric) techniques
Jump-diffusion models

The spot electricity price is assumed to follow a jump-diffusion process:

\[ dS = \mu(S, t)dt + \sigma(S, t)dB + dq(S, t) \]

where \( q(S, t) \) is a compound Poisson process

After a jump the price is forced back to its normal level by mean reversion or

- mean reversion coupled with downward jumps
Modeling average daily Nord Pool prices

Average daily spot prices

Spot Price [NOK/MWh]

Days [30/12/1996-26/03/2000]

(log(Spot Price))

Days [30/12/1996-26/03/2000]
A jump-diffusion model for spot prices

We model the spot price as $p_t = s_t + S_t + \exp(J_t dqq_t + X_t)$:

- $s_t$ is the weekly seasonal component (i.e. an average week),
- $S_t = A \sin(2\pi(t + B)/365) + Ct$ is the annual seasonal component,
- $J_t dqq_t$ is the jump component with $J_t$ being a random jump size, e.g. a lognormal r.v. $\log J_t \sim N(\mu,\rho^2)$, and $q_t$ is a Poisson r.v. with intensity $\kappa$
- $X_t$ is a generalized Ornstein-Uhlenbeck type process:

$$dX_t = (\alpha - \beta X_t)dt + \sigma dB_t = \beta(\frac{\alpha}{\beta} - X_t)dt + \sigma dB_t$$
The non-spike part is Gaussian, spikes exhibit a power-law!
(Markov) Regime-switching models for the spot price

The switching mechanism can be governed by a r.v. that follows a Markov chain with two possible states

A two-state regime model: \( X_t = \{1,2\} \)

\[ P = (p_{ij}) = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} \]
Regime-switching models for the spot price cont.

- Estimation using the EM algorithm

![2-regime model](image1)

![3-regime model](image2)
# Regime-switching models: calibration results

<table>
<thead>
<tr>
<th>Regime Parameter Estimates</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>( c_i )</td>
</tr>
<tr>
<td>--------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td><strong>Two-regime model with Gaussian spikes</strong></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>0.0427</td>
</tr>
<tr>
<td>Spike</td>
<td>—</td>
</tr>
</tbody>
</table>

\[
dY_{t,1} = (c_1 - \beta Y_{t,1})dt + \sigma_1 dW_t \\
Y_{t,2} \sim N(c_2, \sigma_2^2),
\]

| **Two-regime model with lognormal spikes** |
| Base                     | 0.0426    | 0.2078          | 0.0018           | 4.8807           | 0.0217  | 0.9800          | 0.9485 |
| Spike                    | —         | **1.6018**      | **0.0024**       | 4.9678           | 0.0600  | 0.6325          | 0.0515 |

\[
dY_{t,1} = (c_1 - \beta Y_{t,1})dt + \sigma_1 dW_t \\
\log(Y_{t,2}) \sim N(c_2, \sigma_2^2),
\]

| **Two-regime model with Pareto spikes** |
| Base                     | 0.0427    | 0.2087          | 0.0020           | 4.8837           | 0.0231  | 0.9842          | 0.9664 |
| Spike                    | —         | **6.6848**      | **4.2382**       | 4.9837           | 0.7931  | 0.5464          | 0.0336 |

\[
dY_{t,1} = (c_1 - \beta Y_{t,1})dt + \sigma_1 dW_t \\
Y_{t,2} \sim F_{\text{Pareto}}(c_2, \sigma_2^2) = 1 - \left( \frac{c_2}{x} \right)^{\sigma_2^2}
\]
Regime-switching models: spikes

<table>
<thead>
<tr>
<th></th>
<th># spikes</th>
<th>$v_{0.99}$</th>
<th>$v_{0.995}$</th>
<th>max</th>
<th>min</th>
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<tr>
<td>Data ($d_t$)</td>
<td>9.00</td>
<td>0.1628</td>
<td>0.2235</td>
<td>1.1167</td>
<td>−0.7469</td>
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<tr>
<td>2-regime (normal)</td>
<td>17.26</td>
<td>0.3310</td>
<td>0.4523</td>
<td>0.7580</td>
<td>−0.8038</td>
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<tr>
<td>2-regime (log-normal)</td>
<td>18.05</td>
<td>0.3353</td>
<td>0.4648</td>
<td>0.7937</td>
<td>−0.7875</td>
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<tr>
<td>2-regime (Pareto)</td>
<td>33.32</td>
<td>0.5410</td>
<td>0.7851</td>
<td>2.1688</td>
<td>−2.2602</td>
</tr>
</tbody>
</table>

- **Spike** is defined as a change in log-prices greater than 30%
- Number of extreme events is overestimated in all models
- Spike sizes are underestimated for normal and log-normal, but overestimated for Pareto
- Tail exponent (Pareto) is above 4, perhaps the estimation procedure yields a relatively too large scaling parameter
Problems and improvements

- Spike intensity is not time homogenous
  - NHPP in jump-diffusion models
  - Spike size – price level correlation
- Fundamentals are not taken into account
  - Hybrid models – fuel prices, planned and forced outages, generation stack, weather (temperatures), demand function, …

- There’s plenty of work for everyone!