Forecasting spot electricity prices with time series models

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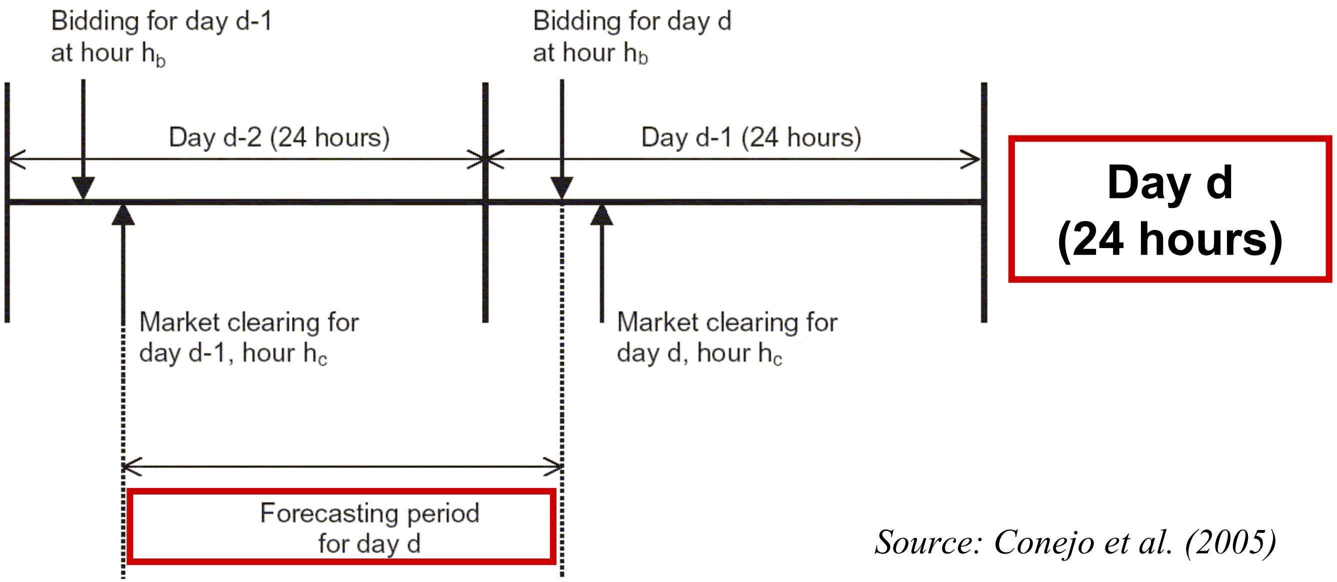
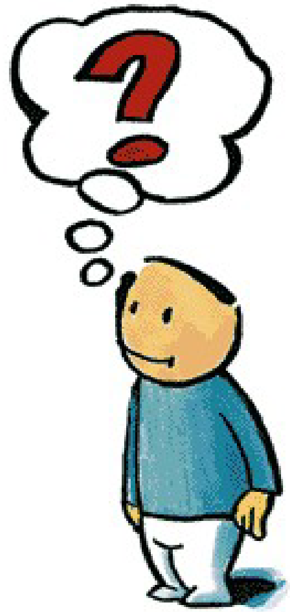
What is the question?

- Even accurate load forecasts cannot guarantee profits in deregulated markets
- Market risk is the outcome of volume and price risk
- **How can we forecast electricity prices?**

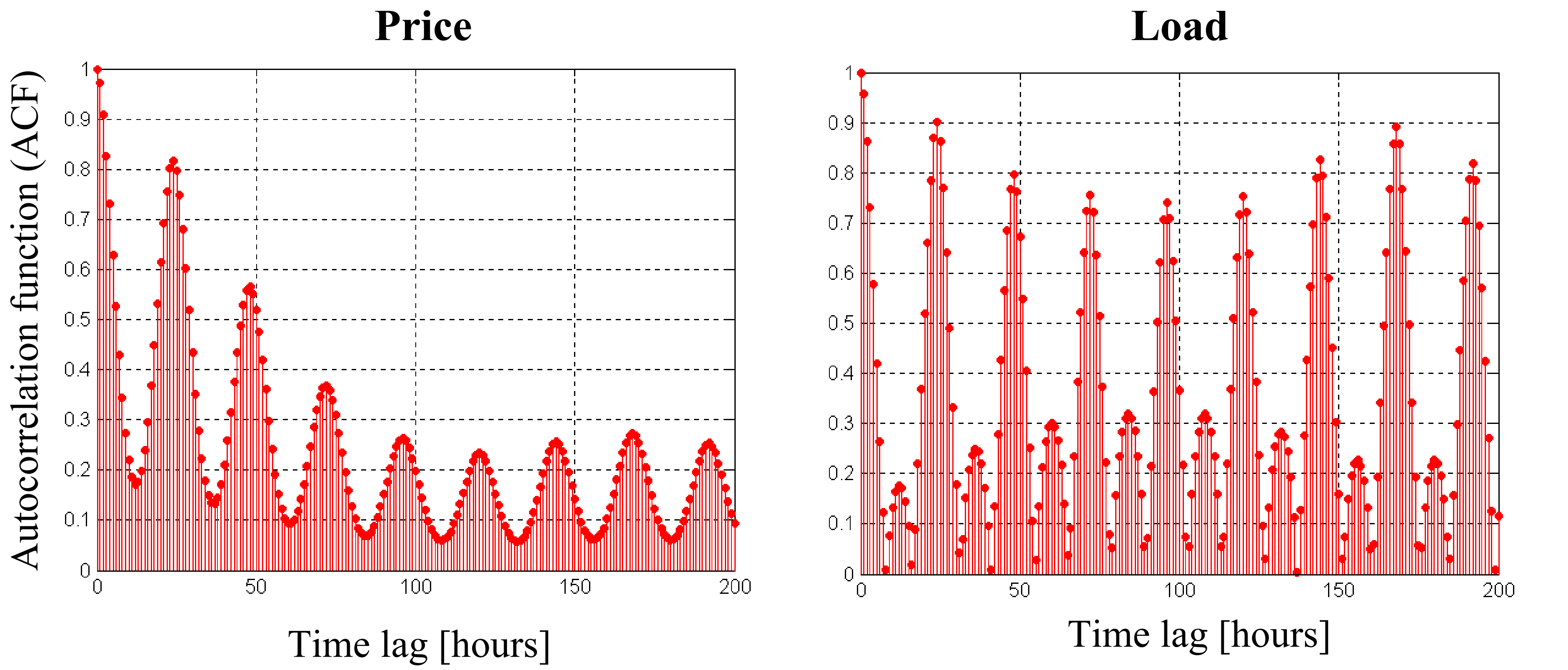


Source: Conejo et al. (2005)



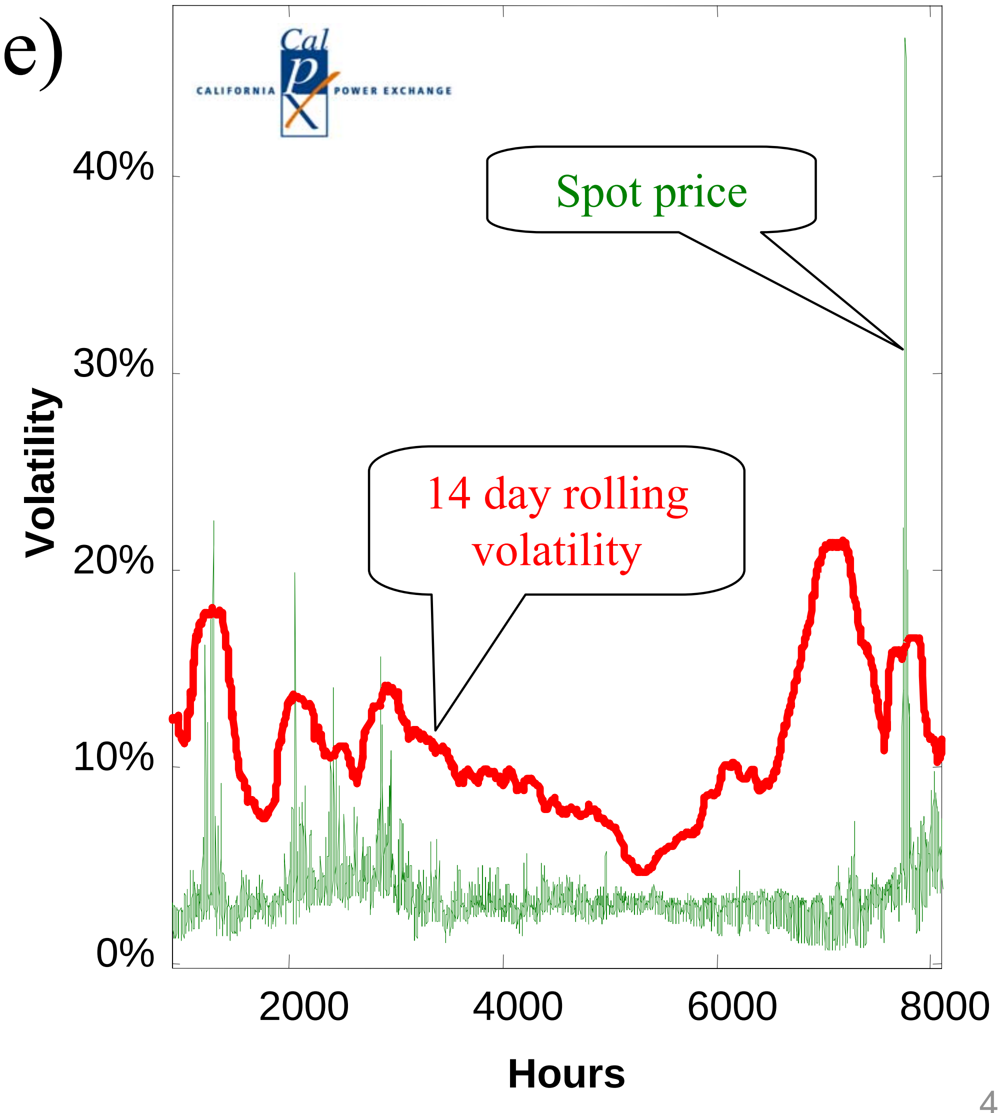
Properties of spot prices - seasonality

- Seasonality at the daily, weekly and annual level
- However, not as **pronounced** as for loads



Properties of spot prices - volatility

- **Extreme** volatility (variance)
- As much as 50% on the daily scale
- Mostly due to seasonality and price spikes
- **Heteroscedasticity**, i.e. time changing volatility



Properties of spot prices - exogenous variables

- Many **quantifiable** exogenous variables – loads, weather, network constraints, etc.

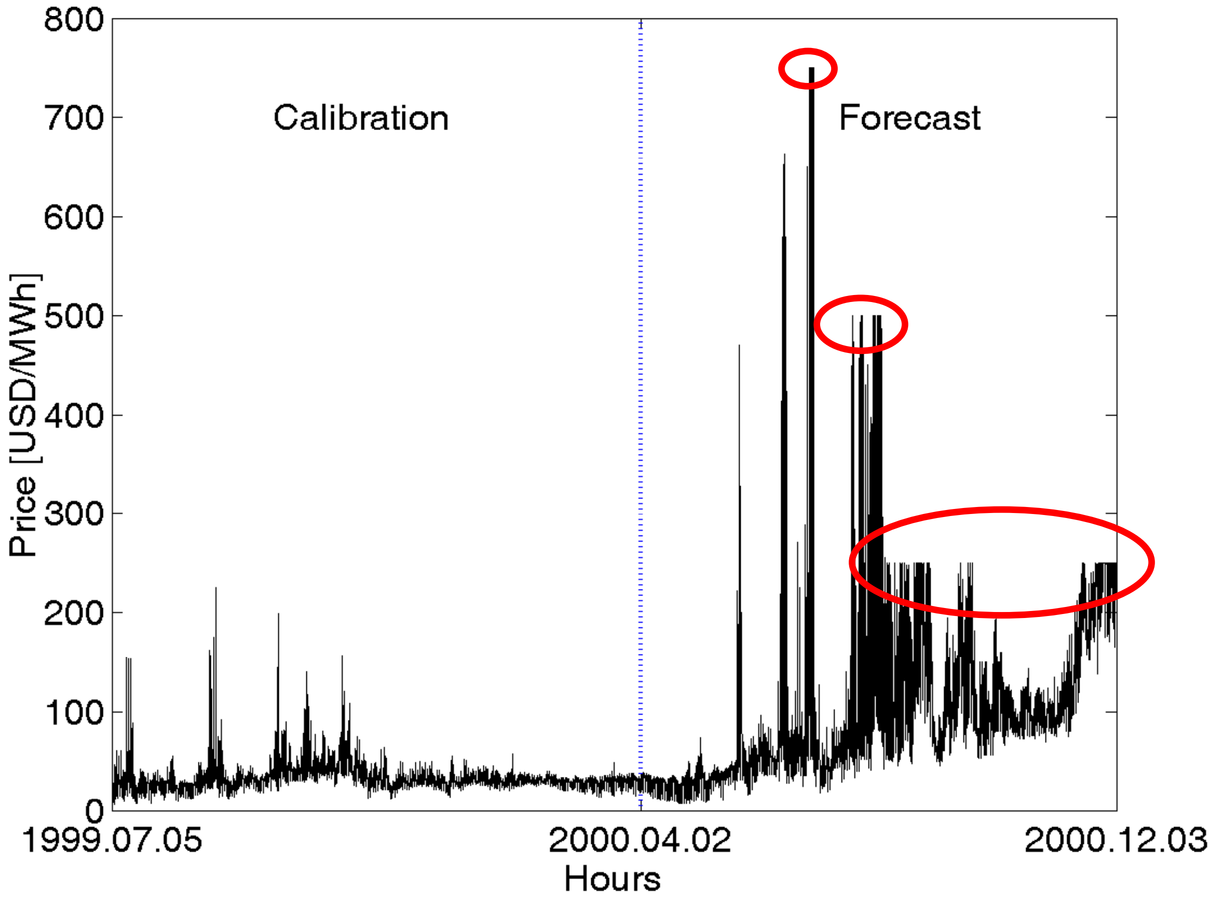
- Psycho- and sociological factors
- Social validation – look at what others are doing – can lead to **market panic**

Forecasting approaches

- Parsimonious stochastic models
  - Finance-inspired stochastic models, e.g. mean-reverting jump-diffusion; ultimate objective – **derivatives valuation**
  - Time series models: ARMA, ARMAX, (S)ARIMA, GARCH
- Structural or fundamental models
  - e.g. model hydrological inflow and snow-pack development
- Non-parametric models
  - ANN, fuzzy systems, genetic algorithms, SVM
  - not intuitive, no simple physical interpretation

Analyzed data

- California Power Exchange (CalPX) hourly market clearing prices
- Why CalPX data?
  - Freely available!
  - Often studied in the literature
  - Extreme market behavior (**crash**)



Studied models

- **Naïve - similar day approach**
  - Mo, Sa, Su: week before; other: previous day
- **ARMA**

$$A(p)P_t \equiv P_t - a_1 P_{t-1} - \ldots - a_p P_{t-p} =$$
$$= \varepsilon_t + b_1 \varepsilon_{t-1} + \ldots + b_q \varepsilon_{t-q} \equiv B(q)\varepsilon_t,$$

Price

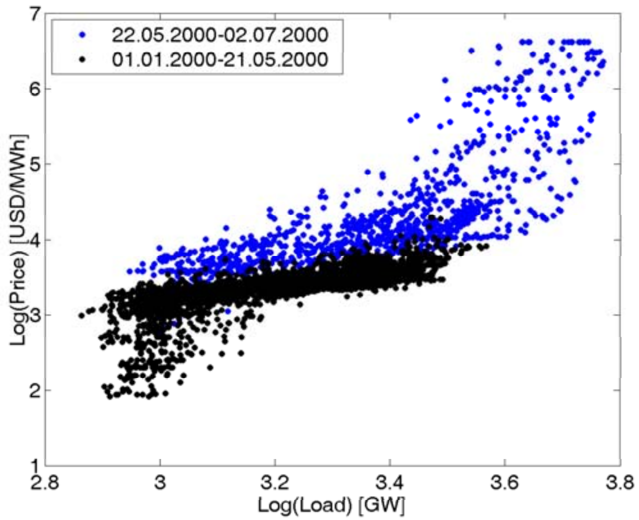
Noise

- **ARMAX**

$$A(p)P_t = C(r,k)Z_t + B(q)\varepsilon_t$$

$$C(r,k)Z_t \equiv Z_{t-k} + c_1 Z_{t-k-1} + \ldots + c_r Z_{t-k-r}$$

Exogenous variable (load)



Model finetuning

- Logarithmic transformation → more **stable variance**
- Mean (or median for loads) removed → **centering**
- Modeling implemented separately across the hours → recovery of each hour's **distinct price profile**
- At lag 0: **CAISO day-ahead load forecast** for larger lags: actual system load
- Large MA part typically **decreases** the performance
- Best results obtained for **pure ARX models**

The optimal model structure

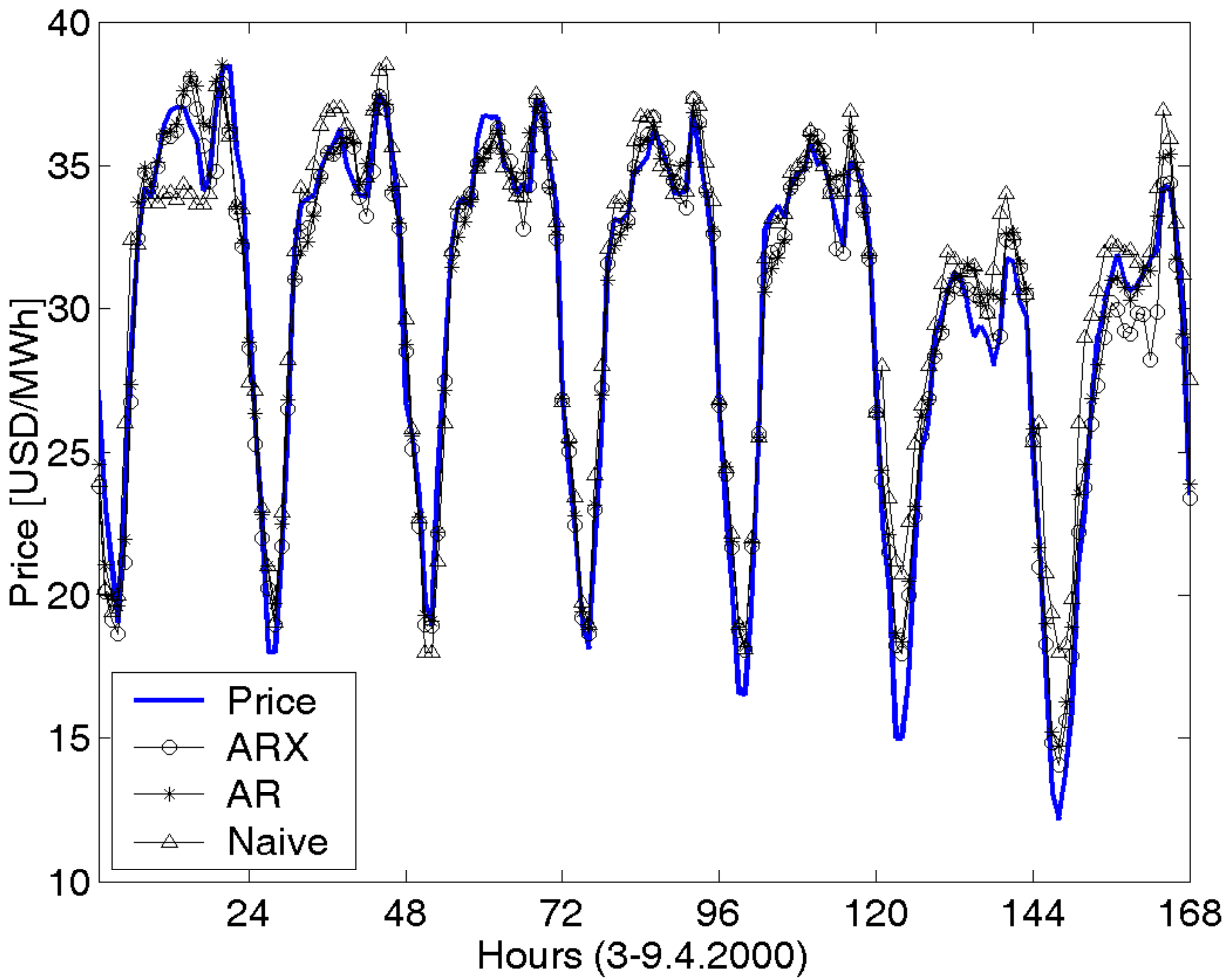
- Inclusion of **3 dummy variables** (for Monday, Saturday and Sunday) helps a lot

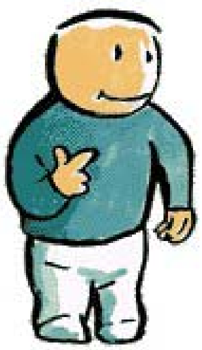
$$p_t - a_1 p_{t-24} - a_2 p_{t-48} - a_3 p_{t-168} - a_4 mp_t =$$

$$= c_1 z_t + d_{Mon} + d_{Sat} + d_{Sun} + \varepsilon_t,$$

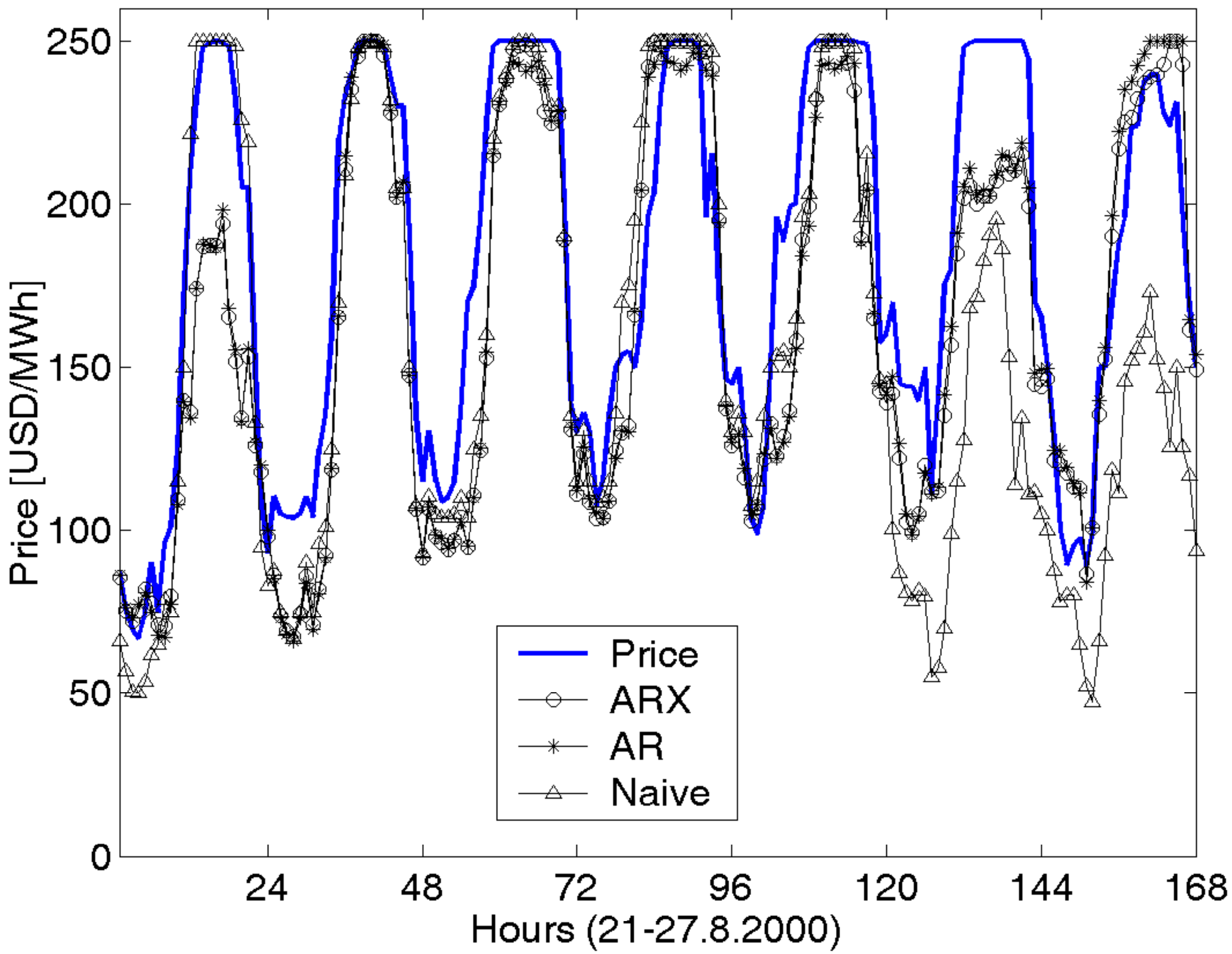
- where $p_t=\log(P_t)$ and
- mp_t is a function of all prices on the previous day (e.g. **minimum** of the 24 hourly prices)

Day-ahead price forecasts for the 1st week

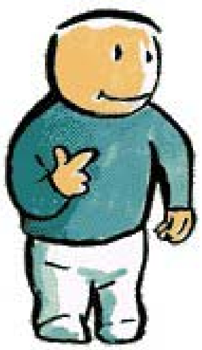


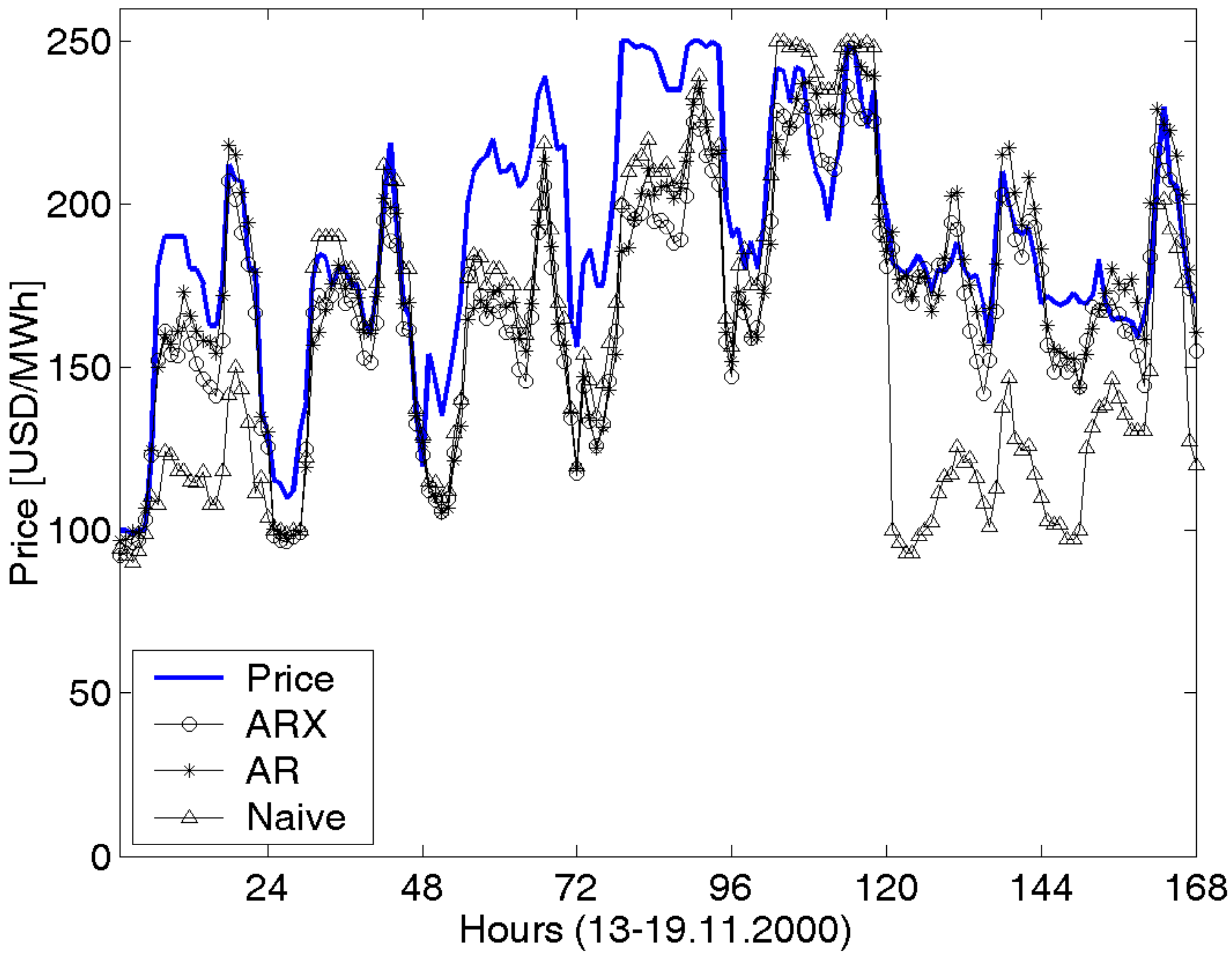


Day-ahead price forecasts for the 21st week



Day-ahead price forecasts for the 33rd week





Day-ahead forecast errors

Mean daily errors (MDE) in percent for the first week of the test period (April 3-9, 2000)

	ARX	AR	Naïve	DR	TF	ARIMA
Mo	3.91	3.73	5.68	**2.6**	2.8	4.35
Tu	**2.33**	2.99	3.77	3.3	3.3	*6.17*
We	**2.06**	*2.28*	2.19	*2.7*	*2.9*	*2.60*
Th	**1.58**	1.94	2.97	1.9	2.2	*2.53*
Fr	*2.92*	*3.62*	2.89	2.5	**2.3**	*3.57*
Sa	3.98	5.39	8.72	3.7	**3.6**	8.46
Su	4.87	3.96	10.11	4.0	**3.7**	7.44

$$MDE = \frac{1}{24}\sum_{h=1}^{24}\frac{\left|p_h - p_h^{est}\right|}{\bar{p}_{24}}$$

DR – dynamic regression (~ARX),
TF – transfer function (~ARMAX), ARIMA
→ one large model for all hours

$$\begin{aligned}
&\left(1-\phi_1 B^1-\phi_2 B^2\right)\\
&\times\left(1-\phi_{23}B^{23}-\phi_{24}B^{24}-\phi_{47}B^{47}-\phi_{48}B^{48}\right.\\
&\qquad\left.-\phi_{72}B^{72}-\phi_{96}B^{96}-\phi_{120}B^{120}-\phi_{144}B^{144}\right)\\
&\left(1-\phi_{167}B^{167}-\phi_{168}B^{168}-\phi_{169}B^{169}-\phi_{192}B^{192}\right)\\
&\times(1-B)(1-B^{24})(1-B^{168})\log p_t\\
&\quad= c+\left(1-\theta_1 B^1-\theta_2 B^2\right)\\
&\qquad\times\left(1-\theta_{24}B^{24}-\theta_{48}B^{48}-\theta_{72}B^{72}-\theta_{96}B^{96}\right)\\
&\qquad\times\left(1-\theta_{144}B^{144}\right)\\
&\qquad\times\left(1-\theta_{168}B^{168}-\theta_{336}B^{336}1-\theta_{504}B^{504}\right)\varepsilon_t.
\end{aligned}$$

Source: Nogales et al. (2002), Contreras et al. (2003)

Day-ahead forecast errors during the crisis

	ARX	AR	Naïve	ARIMA	ARIMA-E
			MWE		
1	**3.04**	3.36	5.00	5.01	5.21
2	**4.71**	5.28	8.62	-	-
21	**13.9**	14.1	18.22	15.65	21.03
33	11.1	**10.6**	18.57	13.60	13.68
			WMSE		
1	**15.2**	16.6	26.8	21.19	21.82
2	**20.7**	22.7	38.0	-	-
21	424	**422**	585	470	675
33	348	**331**	546	393	397

$$MWE = \frac{1}{168}\sum_{h=1}^{168}\frac{\left|p_h - p_h^{est}\right|}{\bar{p}_{168}}$$

$$WMSE = \sqrt{\frac{1}{168}\sum_{h=1}^{168}\left(p_h - p_h^{est}\right)^2}$$

$$\begin{aligned}
&\left(1-\phi_1 B^1-\phi_2 B^2\right)\\
&\times\left(1-\phi_{23}B^{23}-\phi_{24}B^{24}-\phi_{47}B^{47}-\phi_{48}B^{48}\right.\\
&\quad\left.-\phi_{72}B^{72}-\phi_{96}B^{96}-\phi_{120}B^{120}-\phi_{144}B^{144}\right)\\
&\left(1-\phi_{167}B^{167}-\phi_{168}B^{168}-\phi_{169}B^{169}-\phi_{192}B^{192}\right)\\
&\times(1-B)(1-B^{24})(1-B^{168})\log p_t\\
&\quad= c+\left(1-\theta_1 B^1-\theta_2 B^2\right)\\
&\qquad\times\left(1-\theta_{24}B^{24}-\theta_{48}B^{48}-\theta_{72}B^{72}-\theta_{96}B^{96}\right)\\
&\qquad\times\left(1-\theta_{144}B^{144}\right)\\
&\qquad\times\left(1-\theta_{168}B^{168}-\theta_{336}B^{336}1-\theta_{504}B^{504}\right)\varepsilon_t.
\end{aligned}$$

ARIMA-E: ARIMA with an explanatory variable (system load)

Source: Contreras et al. (2003)

Conclusions

- Electricity price forecasting is **possible**
- Accuracy is much (2-3 times) lower than for loads
- Extreme market behavior (significant **deviation from stationarity**) leads to large errors
- Simple models seem to provide **reasonable** forecasts